Fractional scaling of quantum Walks

on Percolation Lattices

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Talk for **QAMF 2010, UBC Vancouver**

preprint: `ArXiv:1006.1283`

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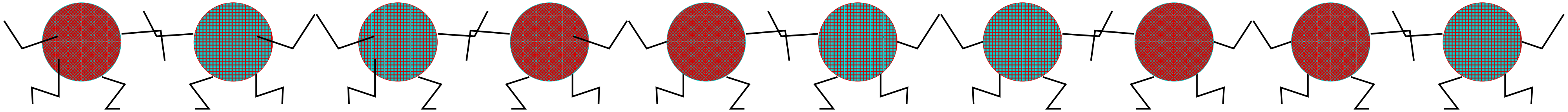
(PK,SC,MT)

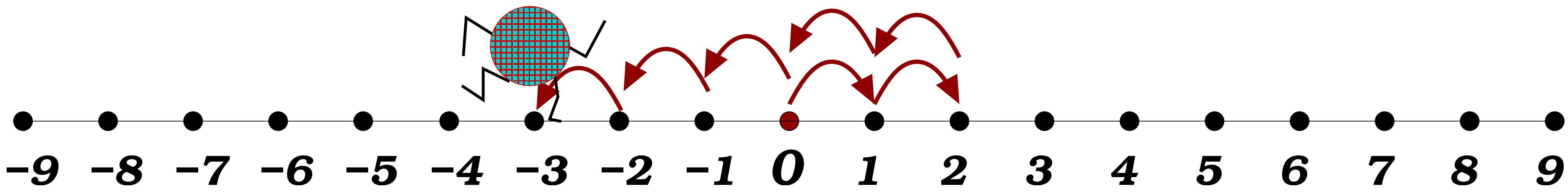
Royal Society University Research Fellowship (VK); Summer bursary (JB)

Overview

1. Introduction & motivation *(quantum walks; percolation lattices)*
2. Quantum walks on line with gaps *(and simple tunnelling model)*
3. 2D percolation lattice *(fractional scaling)*
4. Summary and outlook *(what's next)*



Classical Random Walk on a Line



Recipe:

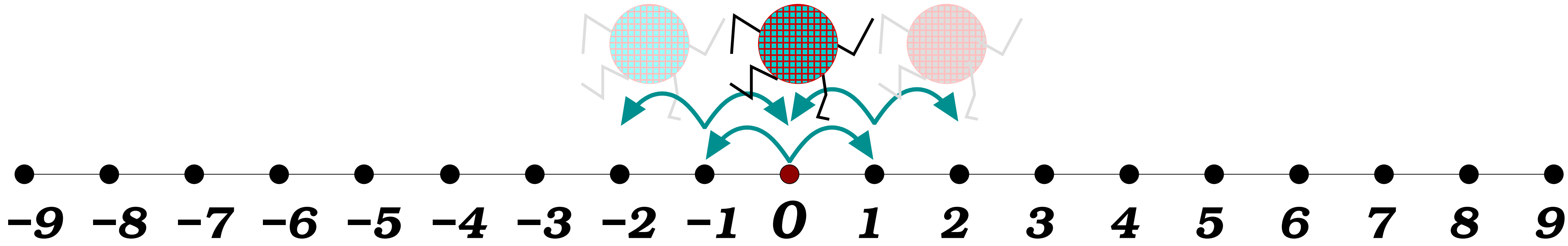
1. Start at the origin
2. Toss a fair coin, result is heads or tails
3. Move one unit: right for heads, left for tails
4. Repeat steps 2. and 3. T times
5. Measure position of walker, $-T \leq x \leq T$

Repeat steps 1. to 5. many times

$\longrightarrow$ prob. dist. $P(x, T)$, binomial

standard deviation $\langle x^2 \rangle^{1/2} = \sqrt{T}$

Quantum Walk on a Line

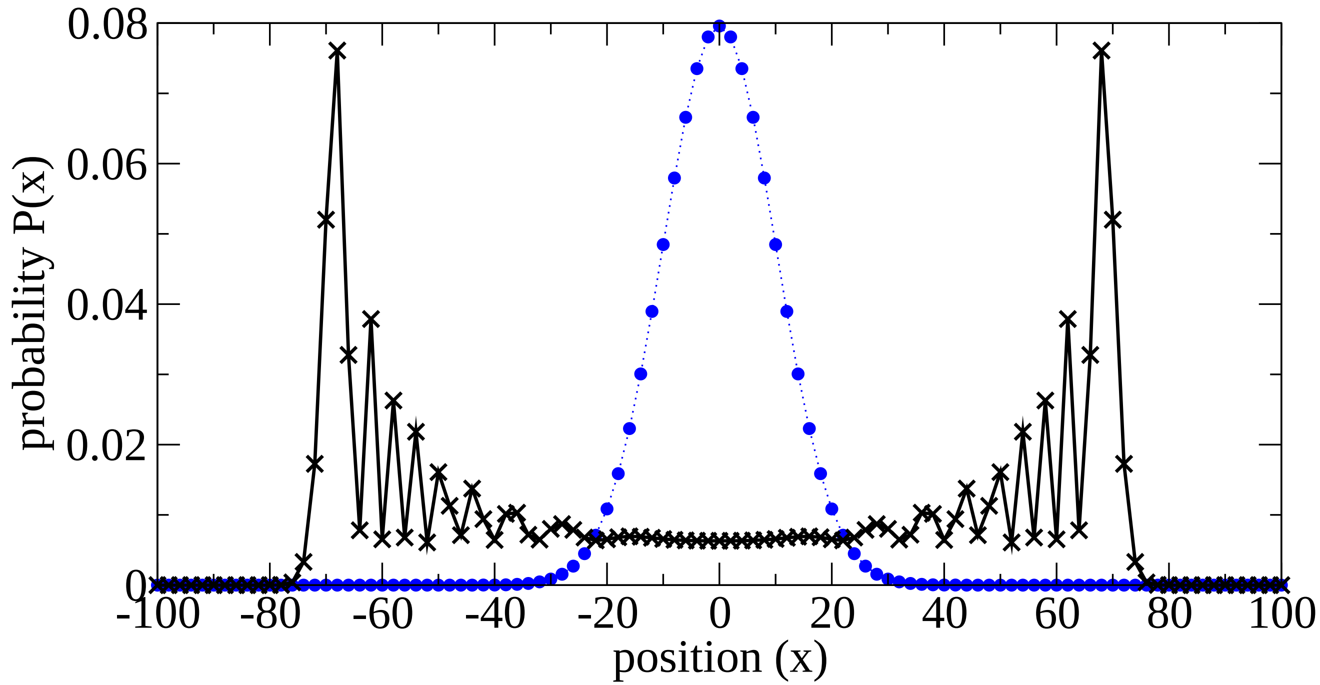


Recipe:

1. Start at the origin
2. Toss a qubit (quantum coin)
   $\mathbf{H}|0\rangle \longrightarrow (|0\rangle + |1\rangle)/\sqrt{2}$
   $\mathbf{H}|1\rangle \longrightarrow (|0\rangle - |1\rangle)/\sqrt{2}$
3. Move <u>left</u> **and** <u>right</u> according to qubit state
   $\mathbf{S}|x, 0\rangle \longrightarrow |x-1, 0\rangle$
   $\mathbf{S}|x, 1\rangle \longrightarrow |x+1, 1\rangle$
4. Repeat steps 2. and 3. T times
5. measure position of walker, $-T \leq x \leq T$

Repeat steps 1. to 5. many times $\longrightarrow$ prob. dist. $P(x, T)$...

Quantum vs Classical on a Line



Analytical solution:

Nayak

Vishwanath

quant-ph/0010117

and

Ambainis

Bach Nayak

Vishwanath Watrous

STOC'01 pp60-69 2001

quantum spread $\propto T$ compared with classical $\sqrt{T}$

Decoherence model for quantum walks

Markovian noise, rate of decoherence: p per unit time
– independent of any previous decoherence events.

Discrete time quantum walk ($\mathbf{S}$ = shift, $\mathbf{C}$ = coin toss):

$$\boldsymbol{\rho}(t+1) = (1-p)\mathbf{S}\mathbf{C}\boldsymbol{\rho}(t)\mathbf{C}^\dagger\mathbf{S}^\dagger + p\sum_i \mathbb{P}_i\mathbf{S}\mathbf{C}\boldsymbol{\rho}(t)\mathbf{C}^\dagger\mathbf{S}^\dagger\mathbb{P}_i^\dagger$$

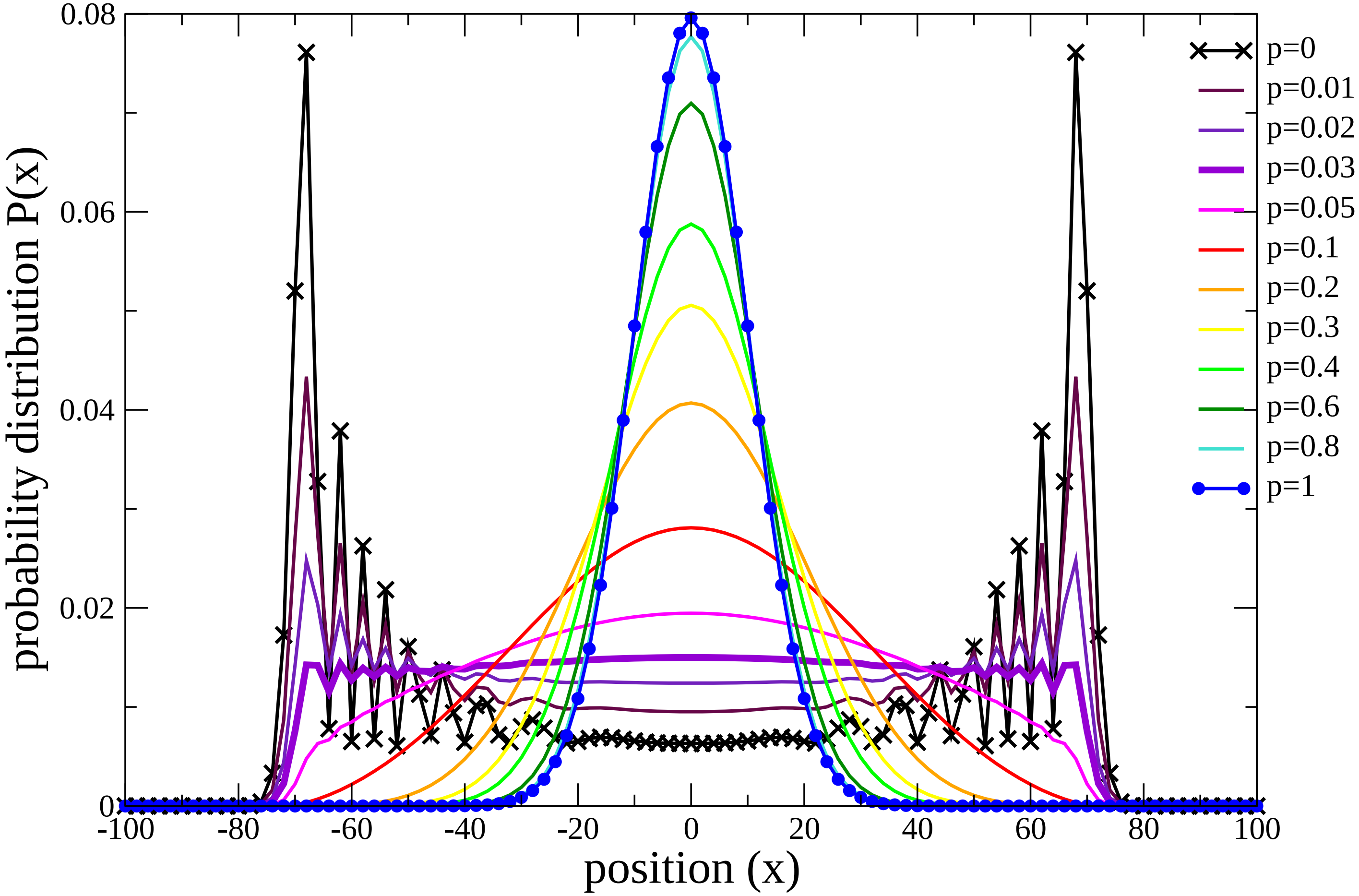
Continuous time quantum walk (hopping rate γ per unit time):

$$\frac{d\boldsymbol{\rho}(t)}{dt} = -i\gamma[\mathbf{A},\boldsymbol{\rho}] - p\boldsymbol{\rho} + p\sum_i \mathbb{P}_i\boldsymbol{\rho}\mathbb{P}_i^\dagger$$

$\longrightarrow$ effect is to reduce size of off-diagonal elements of $\boldsymbol{\rho}$

Interpolate quantum ⟷ classical

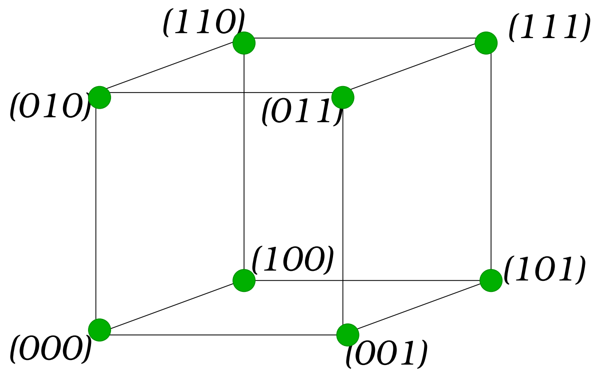
Add decoherence (measure with prob *p* at each step):



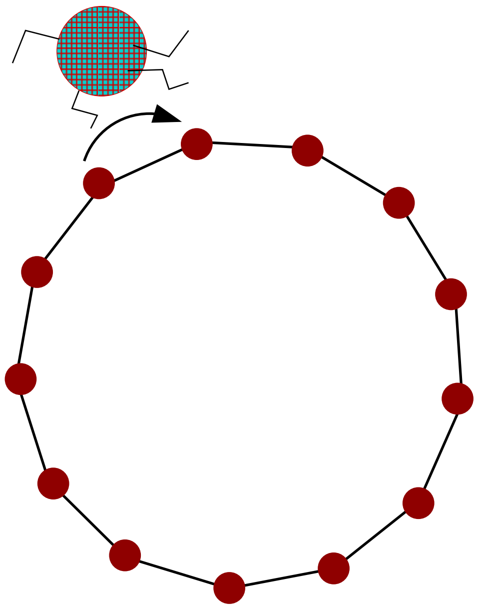
Top hat distribution for just the right amount of noise! `[quant-ph/0209005]`

Beyond the line

Can “quantum walk” on any graph structure:

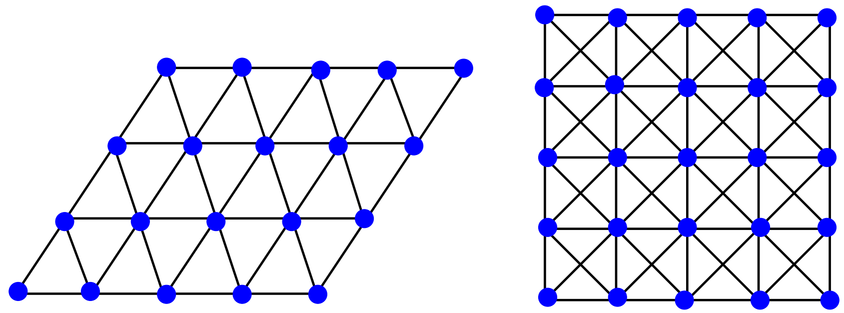


a cycle:



...or lattices:

...or grids:



(a) (b)

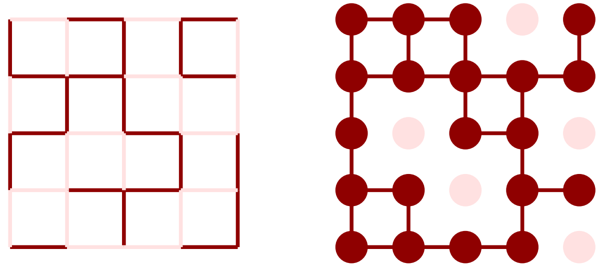
need larger coin: one dimension per edge

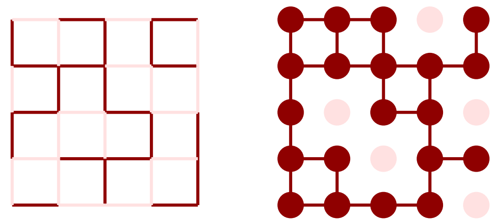
`quant-ph/0304204, quant-ph/0504042`

Motivation:

Role of symmetry in quantum speed up?

- nice properties of quantum walks appear on structures with high symmetry...
- evidence that the symmetry is required:
  – properties disappear when symmetry breaks...
- look at less regular systems: choose **percolation lattices**
- well studied, simple, wide range of applications
- non-trivial phase transition with high upper critical dimension
- disordered but not so random there is no structure





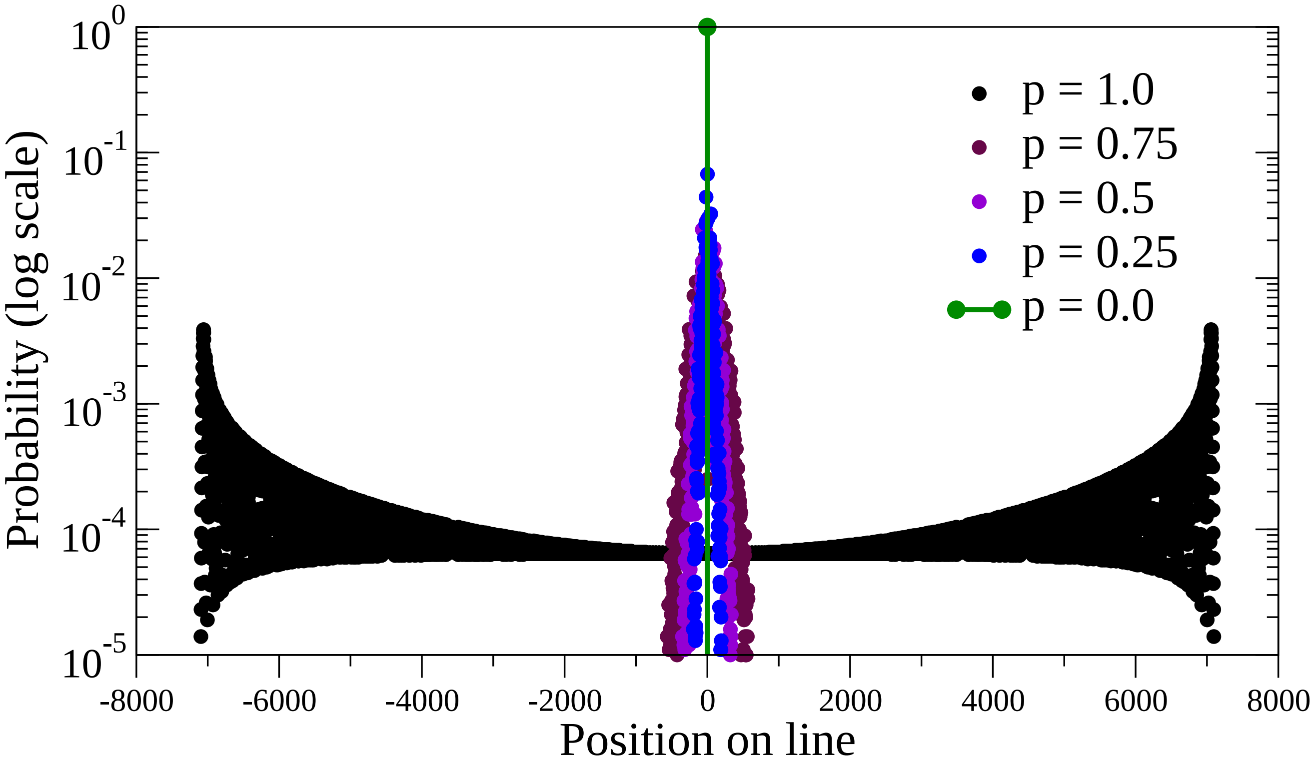
Percolation lattices:

- Two types: bond (edge) and site (vertex)
- bond or site is present in lattice with probability p
- phase transition at $p_c = 0.5$ (bond) or $p_c = 0.5927...$ (site) in 2D lattices
- $p < p_c$: connected structures small
  $p > p_c$: connections link most of lattice, “one giant cluster”, conductivity > 0
- line (1D): no (non-trivial) phase transition:
  single missing bond breaks line into two large structures...

Nonetheless, start with 1D, can still learn something useful.

Percolation on a line

Gaps on a line prevent both classical and quantum walkers from proceeding.



– if gaps change location at each time step: [Romanelli et al quant-ph/0405120] walk still gets through...

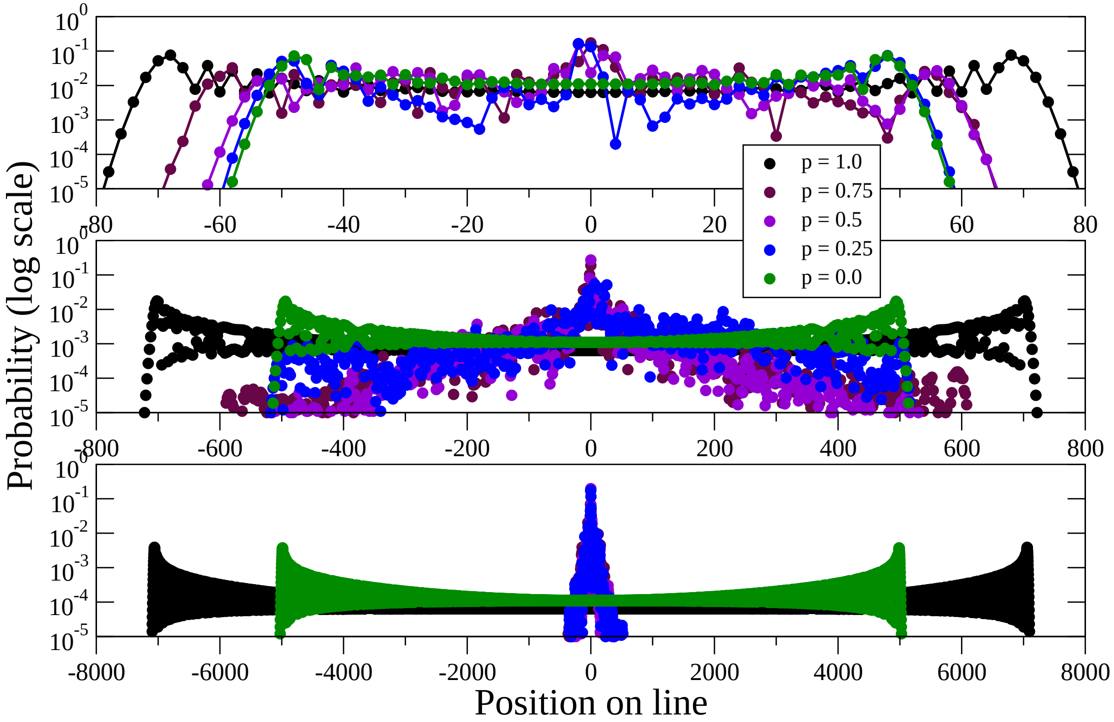
effect of gaps is like decoherence, spreading returns to classical scaling – bit with larger prefactor

Percolation on a line – with tunnelling

alternative: add a simple model of quantum tunnelling:

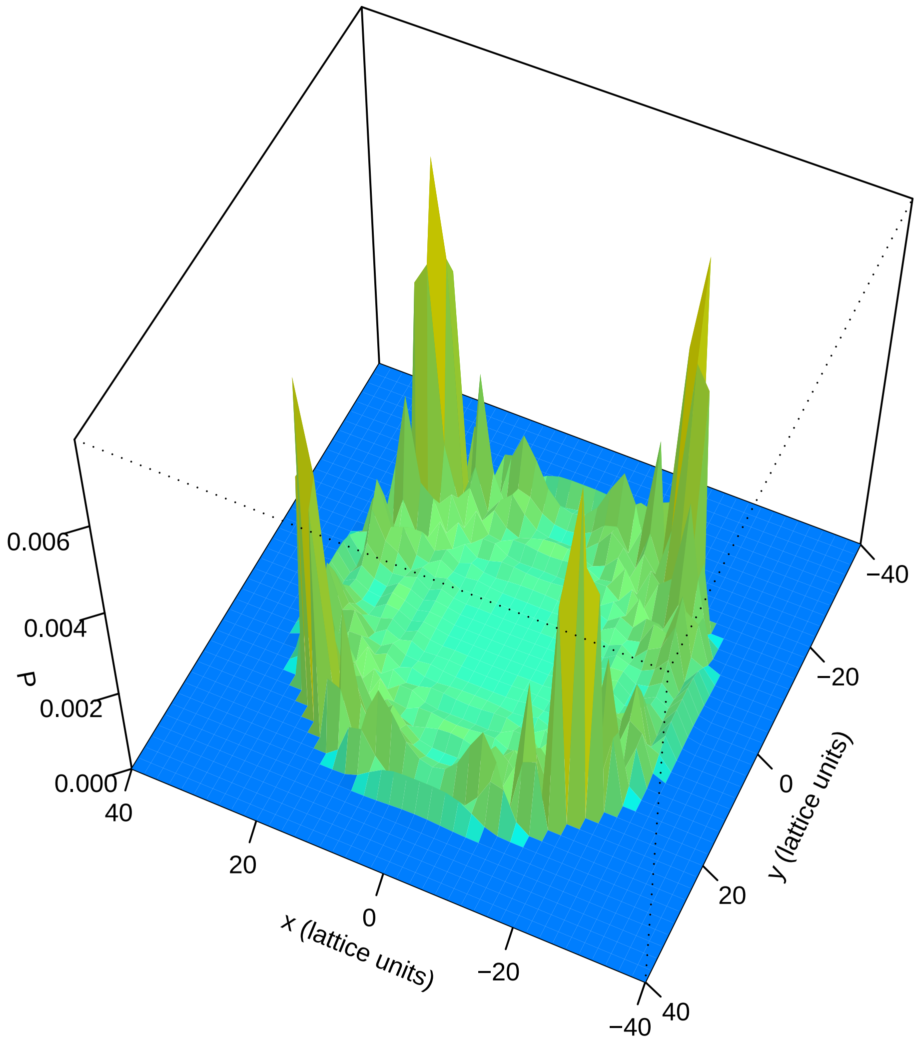
– unbiased coin for edges present ($\eta = 0.25$ below)

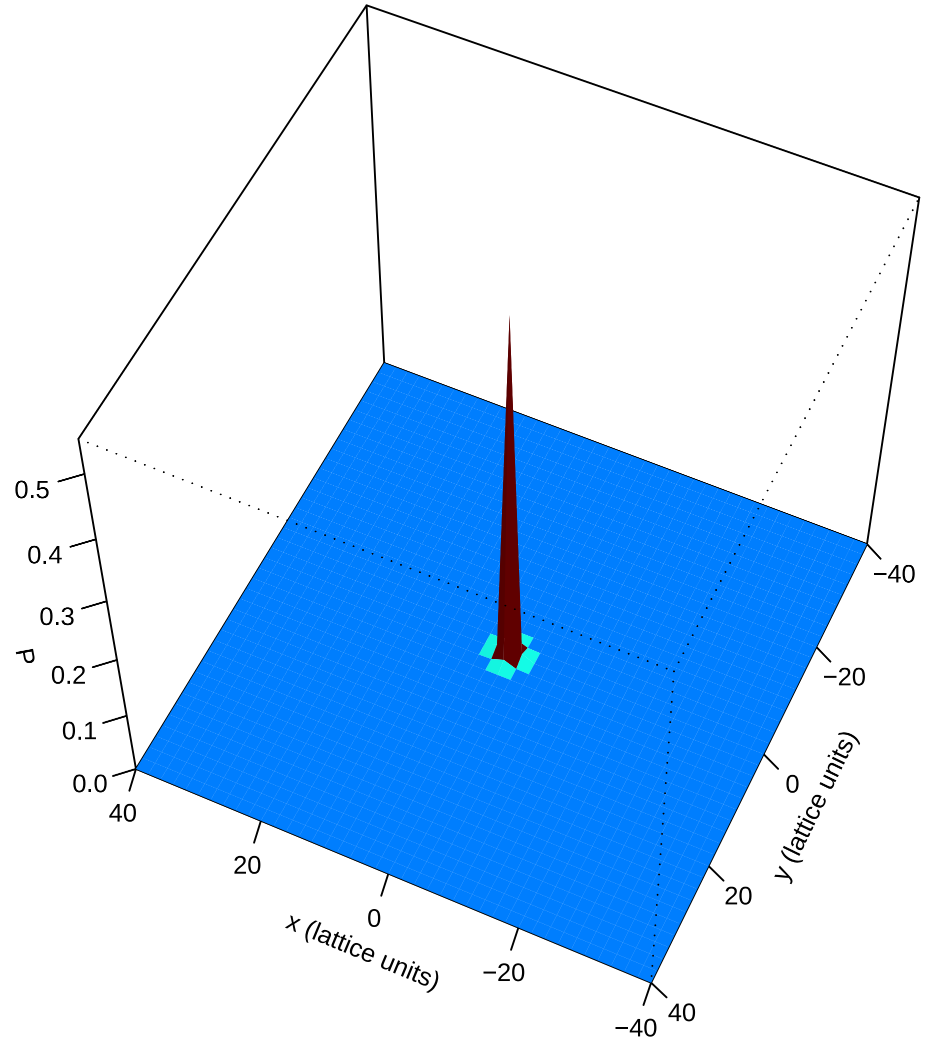
– biased coin allows hopping over missing edges with small probability η



regularly placed gaps equivalent to study by Linden and Sharam arXiv:0906.3692v1
quantum behaviour but widely different spreading rates

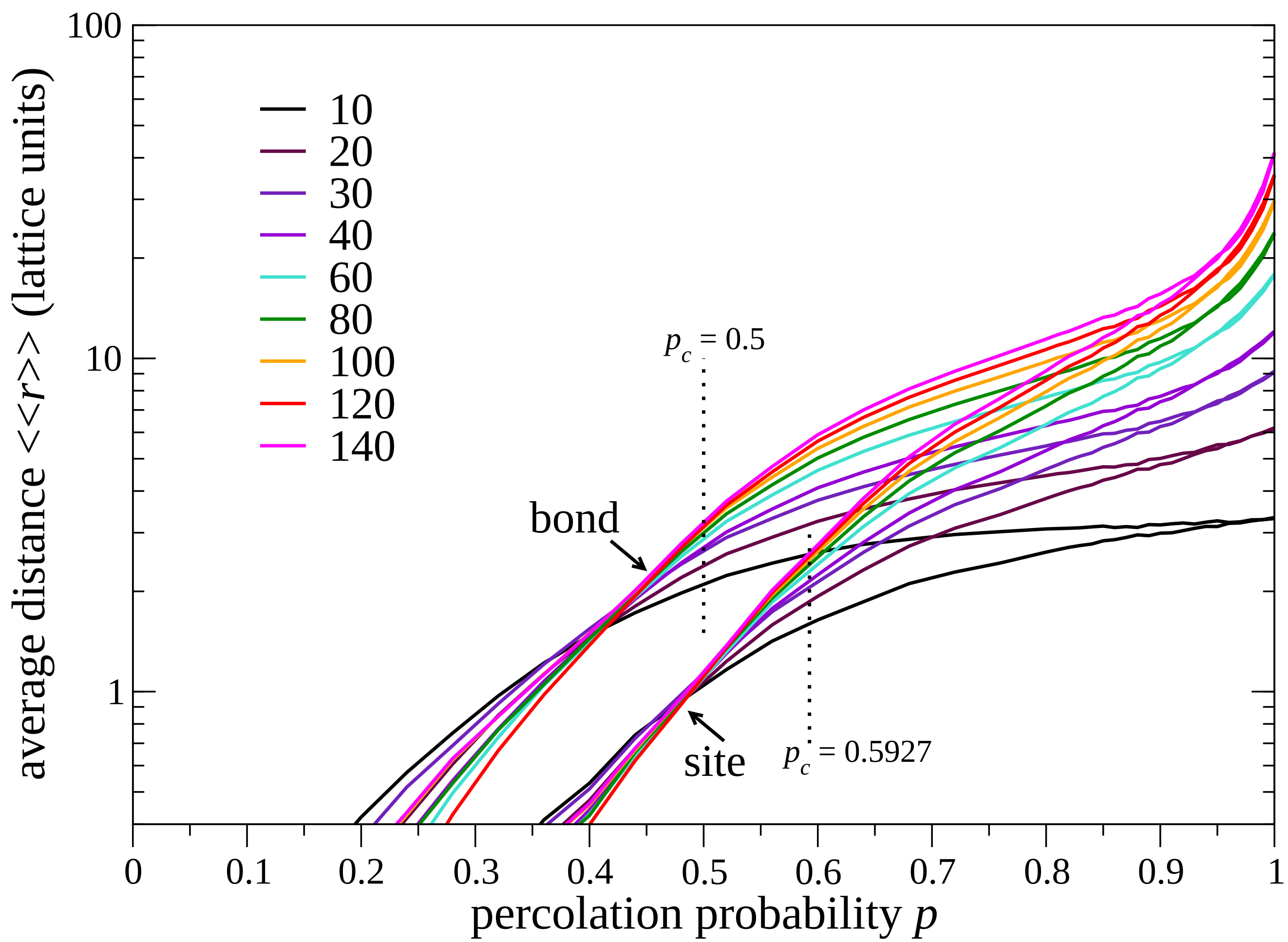
Quantum walk on 2D lattice





- one initial state gives good spreading [Tregenna,Flanagan,Maile,VK, NJP 5 83 (2003)]
- rest give central peak, but intermediate spreading – this is worst case

Quantum walk on 2D percolation lattice

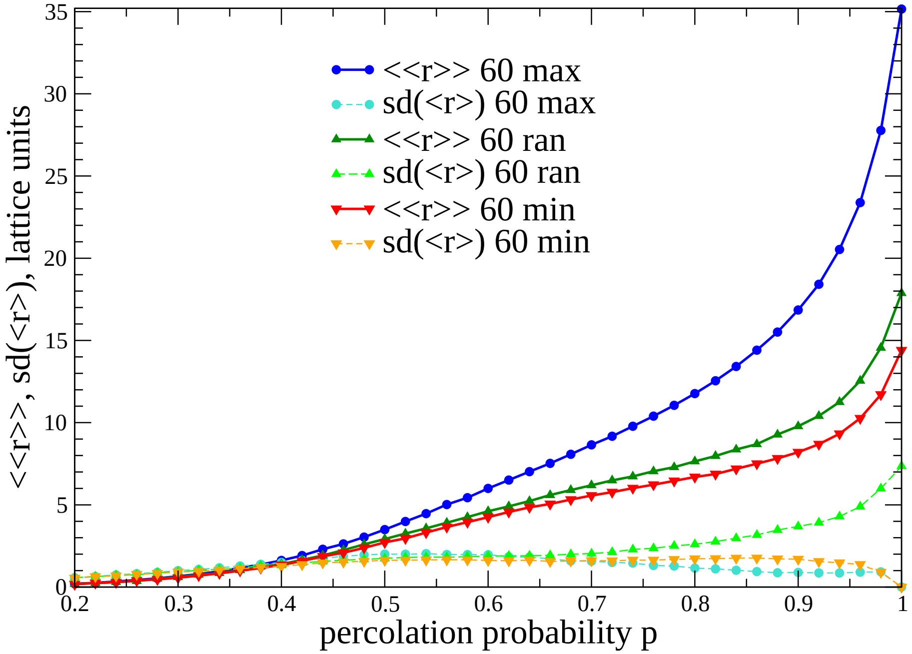


- size in key is number of time steps
- calculate $\langle r \rangle$ for single percolation lattice, then
- average over many (5000) random percolation lattices
- log-linear plot: values of $\langle\langle r \rangle\rangle$ below 1 not important

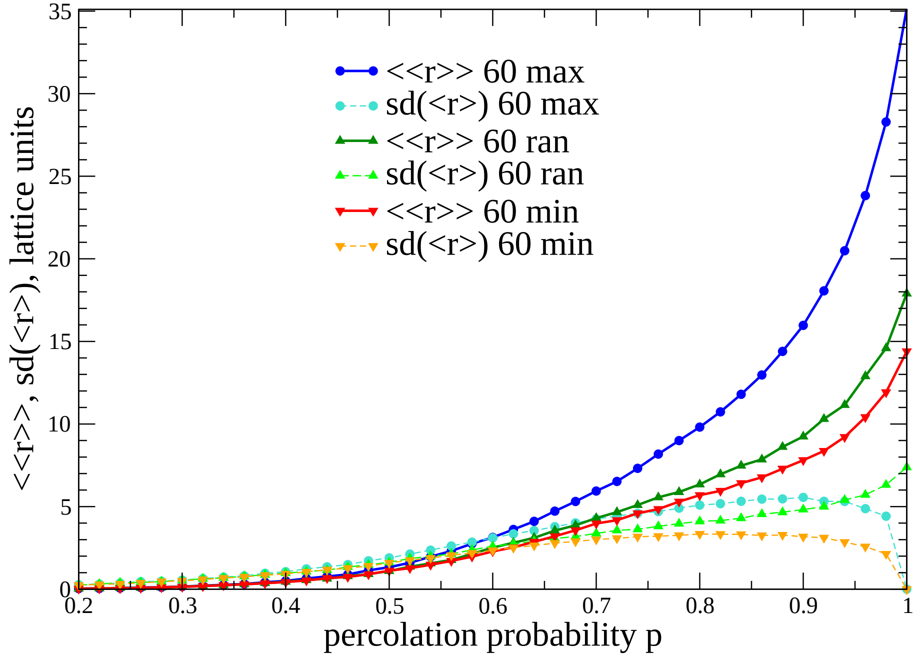
Note finite size effects: 10 $\longrightarrow$ 100 peel off below p_c

Vary initial state:

bond



site

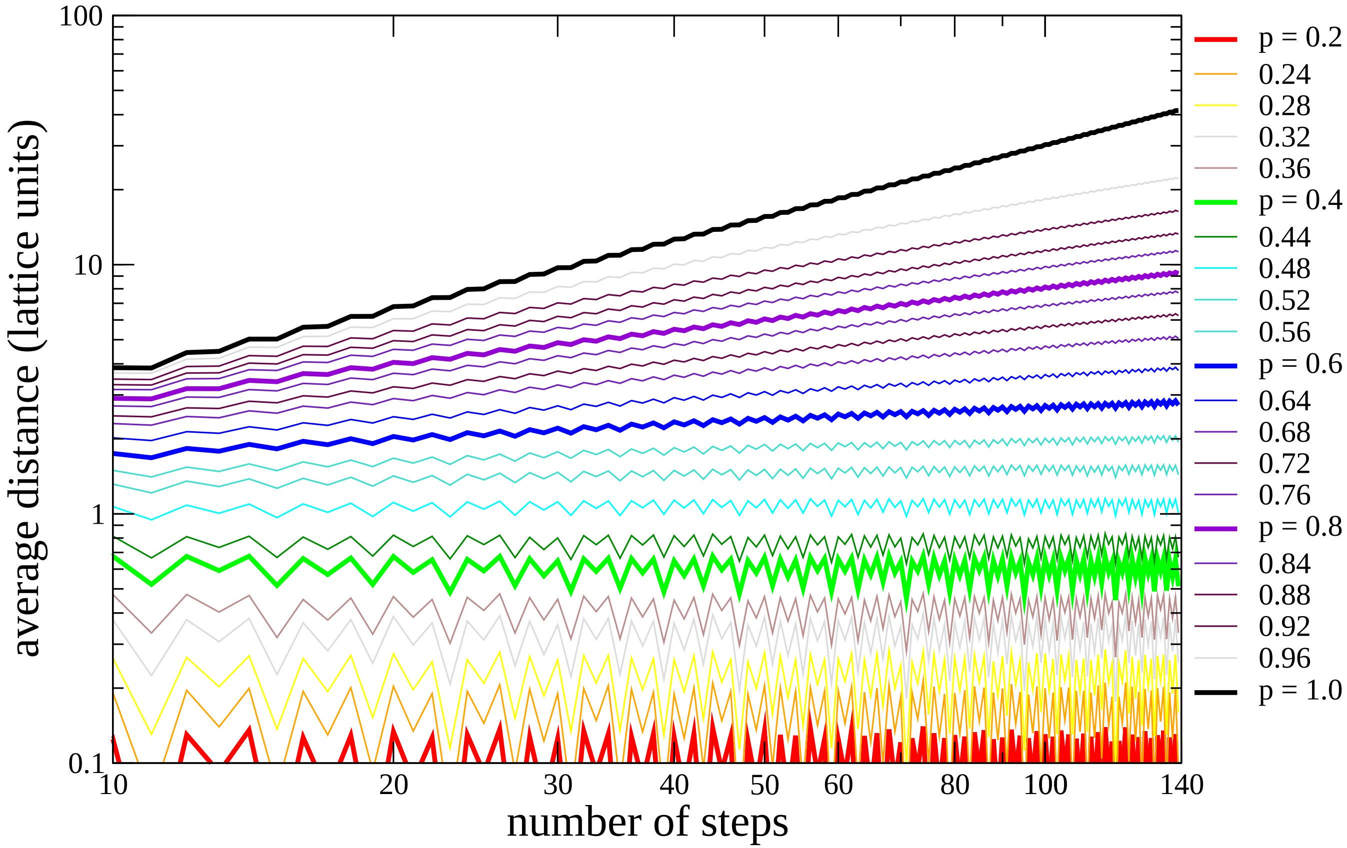


max and min correspond to initial states

ran is random phases in max state

sd($\langle r \rangle$) is variability of $\langle r \rangle$ over percolation lattices – higher for site percolation

Look at scaling of spreading

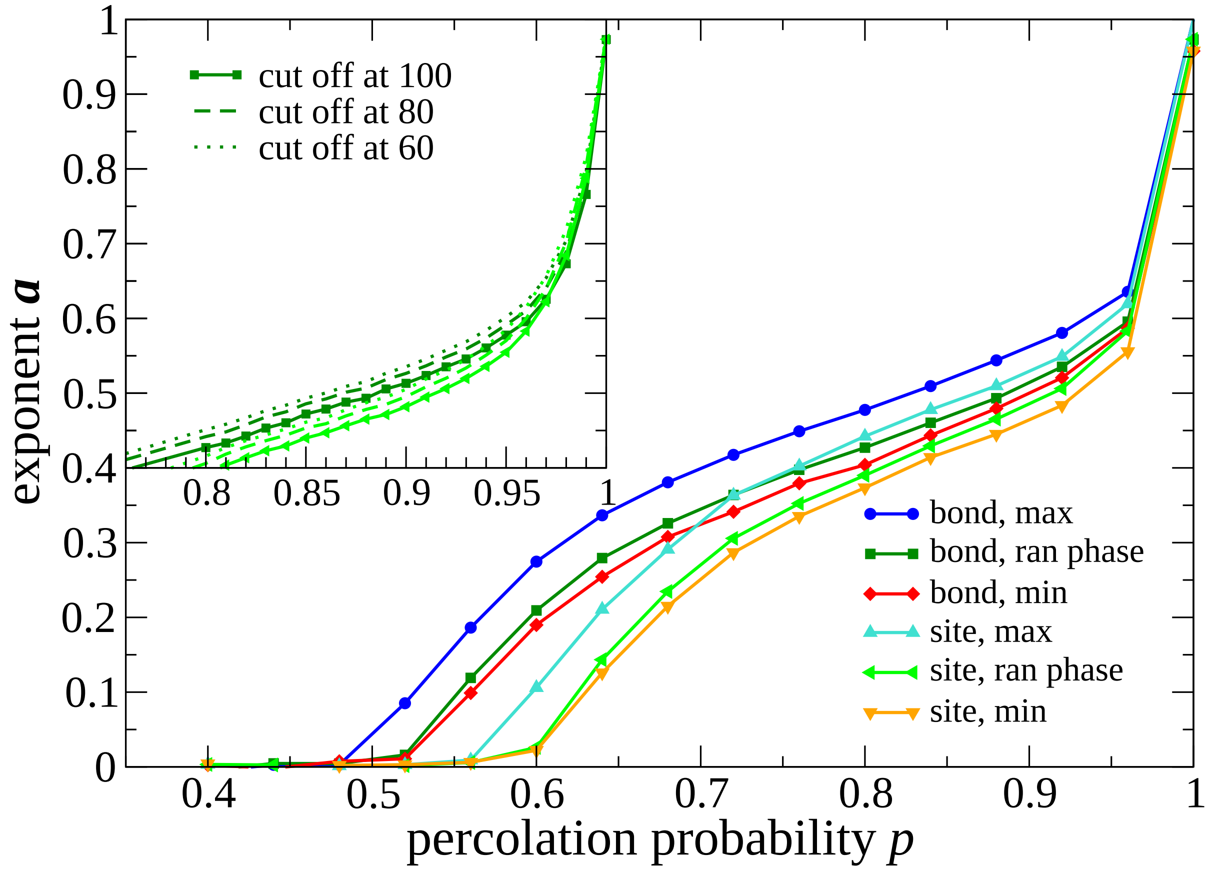


- log-log plot:
slope gives scaling
$\langle\langle r \rangle\rangle \sim T^{\alpha}$

noting finite size effects... use range 100 ⟶ 140 to fit for scaling

Fractional scaling:

fit data in range 100 ⟶ 140 to $\langle\langle r \rangle\rangle \sim T^{\alpha}$

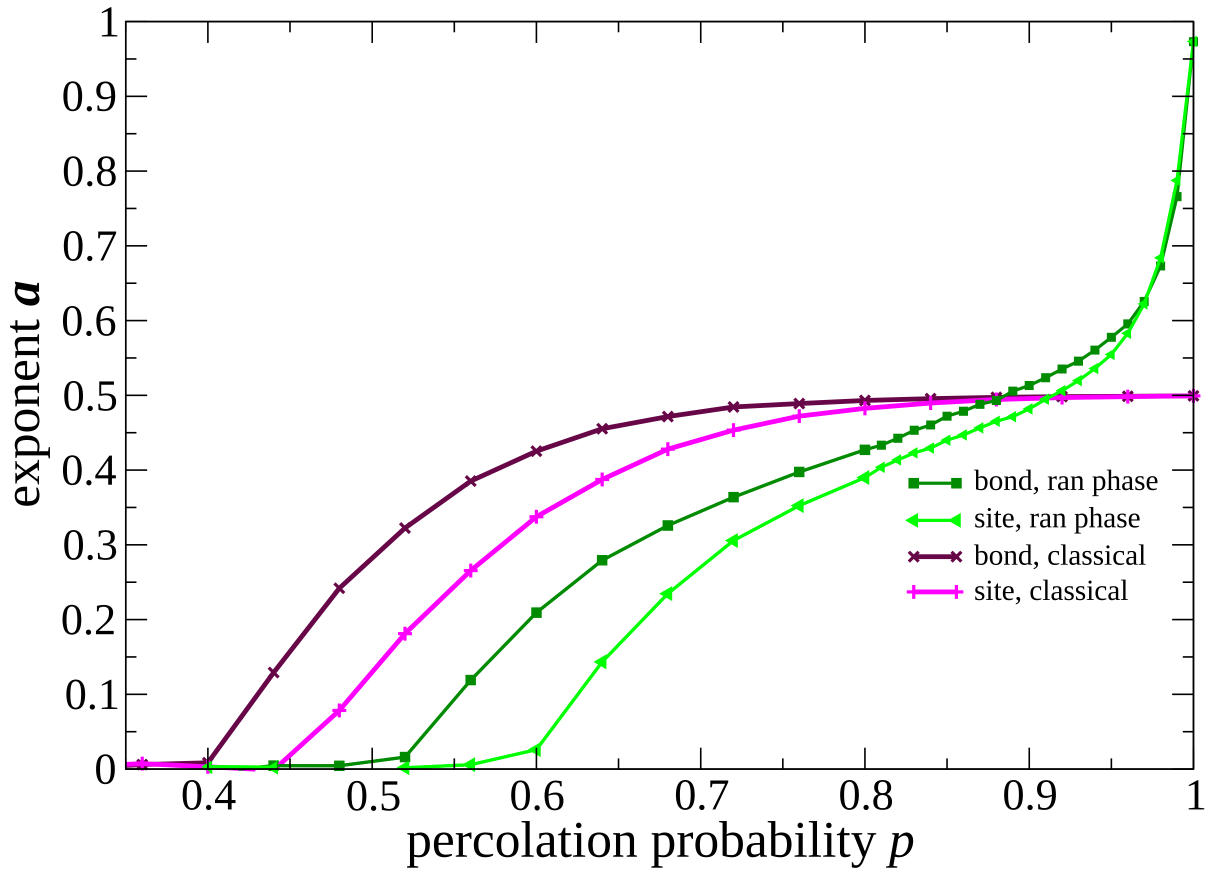


$p_c < p < 0.85$ smooth
rise $0 < \alpha < 0.5$
0.85<p<1.0 steep
rise $0.5 < \alpha \leq 1$

(inset tests finite size convergence using lower cut off for fits)

Compare classical scaling:

– would be nice to know how classical random walk on percolation lattice scales...



same as before (without inset) with classical added... same parameters for fitting, 100 ⟶ 140 still significant finite size effects...

– faster quantum spreading!

– need classical random walk ∼ 900 steps to sample at same rate?

Summary – ArXiv:1006.1283

- classical simulation of quantum walk on 1- and 2-dimensional percolation lattices
- 1D the missing edges induce decoherence at large times
- 2D shows fractional scaling $0 < \alpha \leq 1$ for $p_c < p \leq 1$
- comparison of classical scaling hindered by finite size effects on tractable lattice sizes!
- timescales $\lesssim 10^3$: quantum still dominates?

Further work:

- use percolation lattices to explore properties of quantum walk search algorithm
- can classical simulations of quantum walks do better for analysing percolation lattices?!
- transport, scaling exponents...quantum methods to calculate them?