A Semidefinite Programming Formulation of Quantum and Classical Oracle Problems

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Outline

1. Semidefinite Programming Formulation of Quantum Computation *[2,3]*
   - Can calculate **exact** error probabilities for a quantum computer optimally solving arbitrary quantum oracle problems
2. Remote State Preparation, and a Generalized Objective
3. Restricted Computation
   - Decoherence and analyzing a computation subject to noise
   - Optimal algorithms using reduced quantum resources (briefly)
4. Quantum Clocks

Quantum Algorithms, Generically

Goal: Minimize the number of queries of a quantum query algorithm via a generalization of Ambainis' adversary method *[1] (and recent progress given in [8])*

Try to express a quantum query algorithm as a semidefinite program (SDP):

Semidefinite Programming: Linear programming with semidefinite constraints

Quantum Query Algorithm: Arbitrary unitary operators interspersed with oracle operators acting on some initial state, followed by a measurement

$$|\psi(t)\rangle = U_t \Omega U_{t-1} \Omega \ldots \Omega U_0 |\psi(0)\rangle$$

Grover's Algorithm

Goal: Search – Find the marked element

1. $|\psi(0)\rangle = |0\rangle$
2. $U_0|\psi(0)\rangle = \frac{1}{2}\left(|0\rangle + |1\rangle + |2\rangle + |3\rangle\right)$
3. $\Omega U_0|\psi(0)\rangle = \frac{1}{2}\left((-1)^{\Omega_0}|0\rangle + (-1)^{\Omega_1}|1\rangle + (-1)^{\Omega_2}|2\rangle + (-1)^{\Omega_3}|3\rangle\right)$
4. $\Omega U_0|\psi(0)\rangle = \frac{1}{2}\left(|0\rangle - |1\rangle + |2\rangle + |3\rangle\right)$
5. $U_1\Omega U_0|\psi(0)\rangle = |1\rangle$

U = Inversion about the Mean $a_i \rightarrow -a_i + 2 * \langle a \rangle$

$$\Omega|\psi(t)\rangle = \sum_i a_i \Omega|i\rangle \rightarrow \sum_i a_i (-1)^{\Omega_i}|i\rangle$$

Ω_0	Ω_1	Ω_2	Ω_3
0	1	0	0

Other Oracles Are Possible

We can also have:

$\Omega(0)$	1	0	0	0
$\Omega(1)$	0	1	0	0
$\Omega(2)$	0	0	1	0
$\Omega(3)$	0	0	0	1

Assume each oracle can be given to our computer with some probability

New Goal: Find the marked element $\rightarrow$ Identify the applied oracle

The Computer

$$\rho^Q = \sum_{i,j} a_{i,j} |i\rangle\langle j|$$

Q: Querier system

A: Ancilla

O: Oracle system

$$\rho^O = \sum_{x,y} \sqrt{p_x}\sqrt{p_y}|x\rangle\langle y|$$

Pure state over possible oracles

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$$\Omega^{OQ} = \sum_x |x\rangle\langle x| \Omega^Q(x)$$

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$$\Omega^{OQ}(|x\rangle^O \otimes |\psi\rangle^Q) \rightarrow (|x\rangle^O \otimes \Omega^Q(x)|\psi\rangle^Q)$$

O: Oracle system

$$\rho^O = \sum_{x,y} \sqrt{p_x}\sqrt{p_y}|x\rangle\langle y|$$

Pure state over possible oracles

The Initial Joint Density Matrix

$$\rho^{OQ}(0) = \sum_{x,y} \sum_{i,j} \sqrt{p_x}\sqrt{p_y}|x\rangle\langle y| \otimes a_{i,j}|i\rangle\langle j|$$

$$\rho^{OQ}(0) = \begin{pmatrix} \sqrt{p_0}\sqrt{p_0}\begin{pmatrix} a_{0,0} & a_{0,1} \\ a_{1,0} & \ddots \end{pmatrix} & \sqrt{p_0}\sqrt{p_1}\begin{pmatrix} a_{0,0} & a_{0,1} \\ a_{1,0} & \ddots \end{pmatrix} \\ \sqrt{p_1}\sqrt{p_0}\boxed{\begin{pmatrix} a_{0,0} & a_{0,1} \\ a_{1,0} & \ddots \end{pmatrix}} & \ddots \end{pmatrix}$$

$$\boxed{\begin{pmatrix} a_{0,0} & a_{0,1} \\ a_{1,0} & \ddots \end{pmatrix}} \longrightarrow \rho^{Q}(0)$$

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$$\boxed{\begin{pmatrix} a_{0,0} & a_{0,1} \\ a_{1,0} & \ddots \end{pmatrix}} \rightarrow \rho^{Q}(0)$$

$$\Omega^\dagger\rho^{OQ}(0)\Omega \rightarrow \sum_{x,y}\sum_{i,j}\sqrt{p_x}\sqrt{p_y}|x\rangle\langle y| \otimes \Omega(x)^\dagger a_{i,j}|i\rangle\langle j|\Omega(y)$$

$$\rho^{OQ} = \begin{pmatrix} \sqrt{p_0}\sqrt{p_0}\;\Omega(0)^\dagger\begin{pmatrix} a_{0,0} & a_{0,1} \\ a_{1,0} & \ddots \end{pmatrix}\Omega(0) & \sqrt{p_0}\sqrt{p_1}\;\Omega(0)^\dagger\begin{pmatrix} a_{0,0} & a_{0,1} \\ a_{1,0} & \ddots \end{pmatrix}\Omega(1) \\ \sqrt{p_1}\sqrt{p_0}\;\Omega(1)^\dagger\begin{pmatrix} a_{0,0} & a_{0,1} \\ a_{1,0} & \ddots \end{pmatrix}\Omega(0)^\dagger & \ddots \end{pmatrix}$$

Quantum Algorithm → SDP (1)

Need both an objective and a set of linear constraints over ρ^{OQ}

(Straightforward) **Constraints**

$$\rho^{OQ}(t) \geq 0, \quad \rho^{O}(t) \geq 0$$

$$\rho^{O}(t) = tr_Q(\rho^{OQ}(t))$$

$$\rho^{O}(0) = \rho_0$$

$$t \in \{0 \dots t_f\}$$

Quantum Algorithm → SDP (2)

$$|\psi(t)\rangle = U_t \Omega U_{t-1} \Omega ... \Omega U_0 |\psi(0)\rangle$$

$$\rho^O(t) = tr_{QA}(U_t^{\dagger QA} \Omega^{\dagger OQ} \rho^{OQA}(t-1) \Omega^{OQ} U_t^{QA})$$

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Leads to the SDP S_E, corresponding to the application of oracle and unitary operators as above

$$S_E = \begin{cases} \rho^{OQ}(t) \geq 0, \quad \rho^O(t) \geq 0 \\ \rho^O(t) = tr_Q(\rho^{OQ}(t)) \\ \rho^O(t) = tr_Q(\Omega^{\dagger OQ} \rho^{OQ}(t-1) \Omega^{OQ}) \\ \rho^O(0) = \rho_0 \\ t \in \{0 \ldots t_f\} \end{cases}$$

Measurement and Remote State Preparation

The following protocol describes the measurement process at the end of the computation at time t_f. *[6]*

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Q's POVM is unconstrained, but the set of states that can be prepared on O is restricted by: $\sum_i \sigma_i^O = \rho^O(t_f)$

The Complete SDP

$$S_M = \begin{cases} Minimize \ \sum_i tr(\sigma_i A_i) \ subject \ to: \\ \forall_i \sigma_i \geq 0 \\ \sum_i \sigma_i^O = \rho^O(t_f) \end{cases}$$

$$S_E = \begin{cases} \rho^{OQ}(t) \geq 0, \quad \rho^O(t) \geq 0 \\ \rho^O(t) = tr_Q(\rho^{OQ}(t)) \\ \rho^O(t) = tr_Q(\Omega^{\dagger OQ} \rho^{OQ}(t-1) \Omega^{OQ}) \\ \rho^O(0) = \rho_0 \\ t \in \{0 \ldots t_f\} \end{cases}$$

$$\mathrm{S} = \mathrm{S}_M \cup S_E$$

Conventional Cost Function

- Each measurement outcome *i* corresponds to the querier's guess of which oracle was applied
  - Search: *i = 2* – The marked element is in position 2

$$(\sigma_i)_{x,x} = P(measured\ i \cap oracle\ = \ x)$$

- Average Probability of Error: Probability that you had oracle *x*, measured outcome *i*, and that you shouldn't measure outcome *i* on oracle *x*
  - Search: *x = 1000, i = 2*

$$\epsilon = \sum_i \sum_{x:x\neq i} (\sigma_i)_{x,x} \qquad \langle x|A_i|x\rangle = 1 - \delta(i,x)$$

- This error probability will be **exact**. Q is free to choose the POVM which optimizes the cost function, and the SDP will find this optimum subject to its constraints.

Restricted Computation

- All experimentally available quantum computers are subject to noise
- Decoherence can reduce or eliminate any quantum advantage.
- All current, general quantum lower bound methods assume perfect quantum computation.
- We would like a way to characterize how “quantum” our device is

Decoherence

After each query, the environment will interact with the computer and will decohere the querier *[11]*

Post Unitary, Pre-Query

$$|\psi(0)\rangle^{OQE} = \sum_{x,i} a^{x}_{x,i} |x\rangle^{O} |i\rangle^{Q} |\epsilon_0\rangle^{E_1}$$

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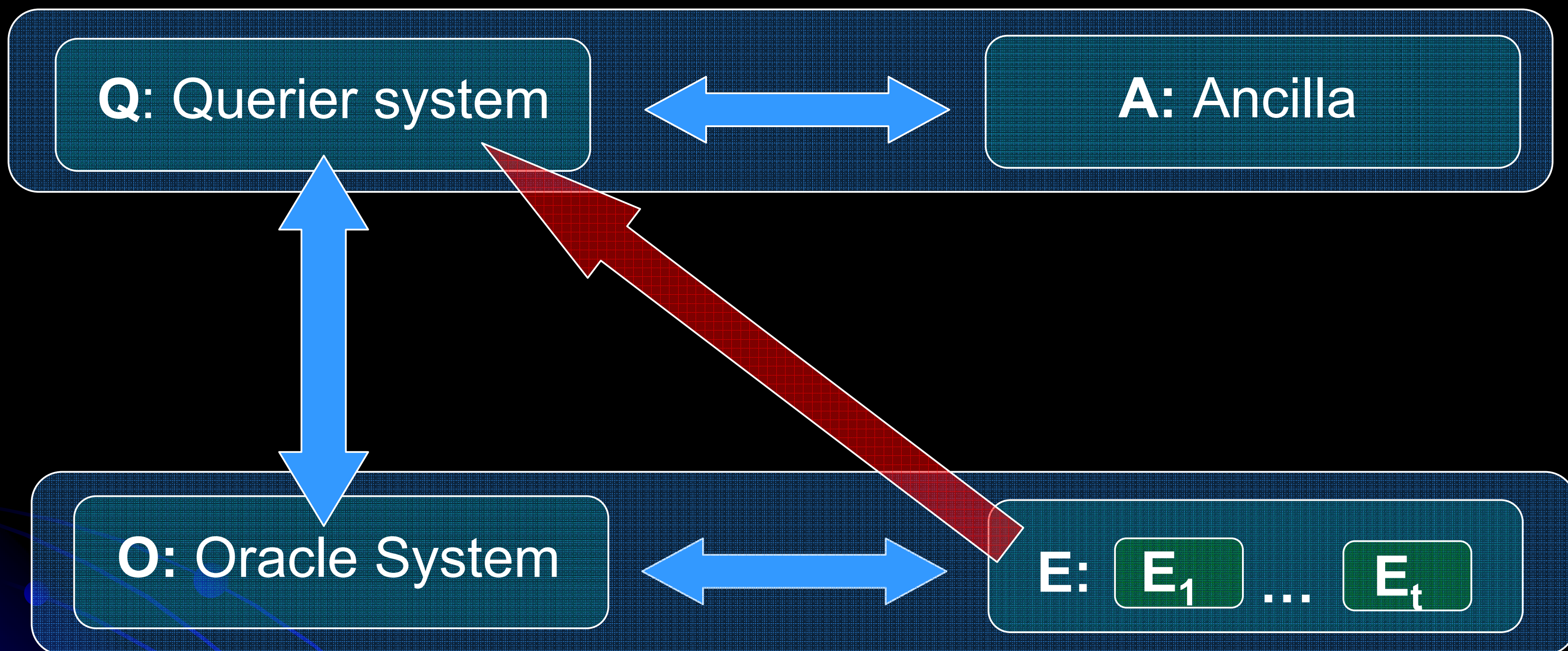
Post Query

$$|\psi(1)\rangle^{OQE} = \sum_{x,i} a^x_{i,a} |x\rangle^O (-1)^{\Omega(x)_i} |i\rangle^Q |\epsilon_i\rangle^{E_1} |\epsilon_0\rangle^{E_2}$$

The size of the environment grows with every query, so at time t:

$$E(t) \equiv E = E_1 \otimes E_2 \ldots \otimes E_t$$

Adding the Environment



The environment is part of the oracle system that the querier has no access to, but acts on the querier during each query.

The Extended SDP

Minimize $tr(\sum_i \sigma_i A_i)$ subject to:

$$\forall_i \sigma_i^{OE} \geq 0$$

$$\rho^{OQE}(t) \geq 0, \quad \rho^{OE}(t) \geq 0$$

$$\sum_i \sigma_i^{OE} = \rho^{OE}(T)$$

$$\rho^{OE}(t) = tr_Q(\rho^{OQE}(t))$$

$$\rho^{OE}(t) = tr_Q(D^{\dagger QE_t}\Omega^{\dagger OQ}\rho^{OQE}(t-1) \otimes |\epsilon_0\rangle_{E_t}^{E_t}\langle\epsilon_0|\Omega^{OQ}D^{QE_t})$$

$$\rho^{OE}(0) = \rho_0$$

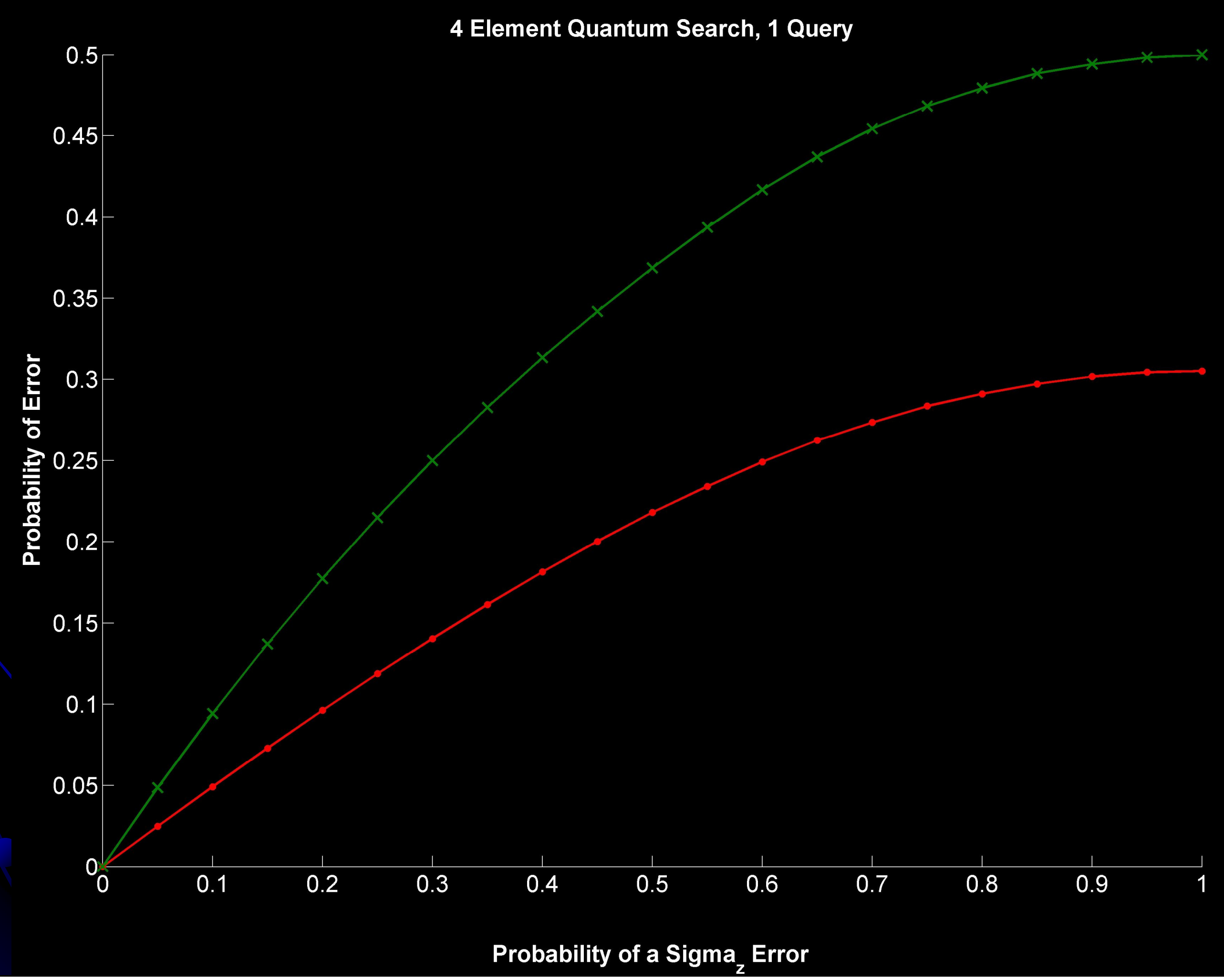
$$t \in \{0 \ldots t_f\}$$

Quantum/Classical Interpolation

One or more qubits may decohere independently with probability *p*

$$|x\rangle^O|i\rangle^Q|\epsilon_0\rangle^E \rightarrow \sqrt{1-p}|x\rangle^O(-1)^{\Omega(x)_i}|i\rangle^Q|\epsilon_0\rangle^E + \sqrt{p}|x\rangle^O(-1)^{\Omega(x)_i}|i\rangle^Q|\epsilon(i)\rangle^E$$

Red: One qubit decoherence
Green: Two qubit decoherence



Comments

- The size of the SDP grows rapidly, particularly when modeling decoherence. (< 10 qubits)
- Since the SDP outputs the optimal density matrix at every time step, we can reconstruct the inter-query unitaries, and hence the optimal algorithm
- We have recently explored these issues from an alternate angle by asking: *Is there an equally optimal algorithm that uses fewer quantum resources?*
  - Use an alternate SDP which tries to split the optimal algorithm into two or more parts that can be classically combined
  - Use the Von Neumann entropy to quantify the quantum resources needed in each branch
- See slides at end of talk for details

Clocks

- Our SDP formulation is highly general, and can describe quantum **processes** as well as quantum query algorithms
- Specifically, if an element drawn from a continuous set of unitary operators, indexed by some parameter, acts on a state, our formulation can be used to optimally estimate the value of this parameter
- We will use this idea to optimize the accuracy of quantum clocks
- **Goal**: Express the operation of the a quantum clock as a black box oracle problem.

Atomic Clock Basics

- Count the ticks of some periodic system and interpret as time.
- Atoms have a discrete set of energy levels and can transition between them by absorbing or emitting photons. ($E = hf$)
- Atomic clocks count the oscillations of a laser whose frequency is stabilized to some atomic transition
- 1 second ≡ 9,192,631,770 cycles of the radiation corresponding to a hyperfine transition in Cesium

NIST F1

Spectroscopy

- Consider a set of *N* ions and the set of states labeled by $|k\rangle$ which have *k* ions in the excited state and *N* - *k* ions in the ground state. (Dicke states)

- Over time, our laser/classical oscillator will drift from the atomic resonance. We need to measure and correct this detuning.

- After an interrogation period of length T with a laser tuned near resonance *[7]*

$$|k\rangle \rightarrow e^{-ik(f-f_0)T}|k\rangle$$

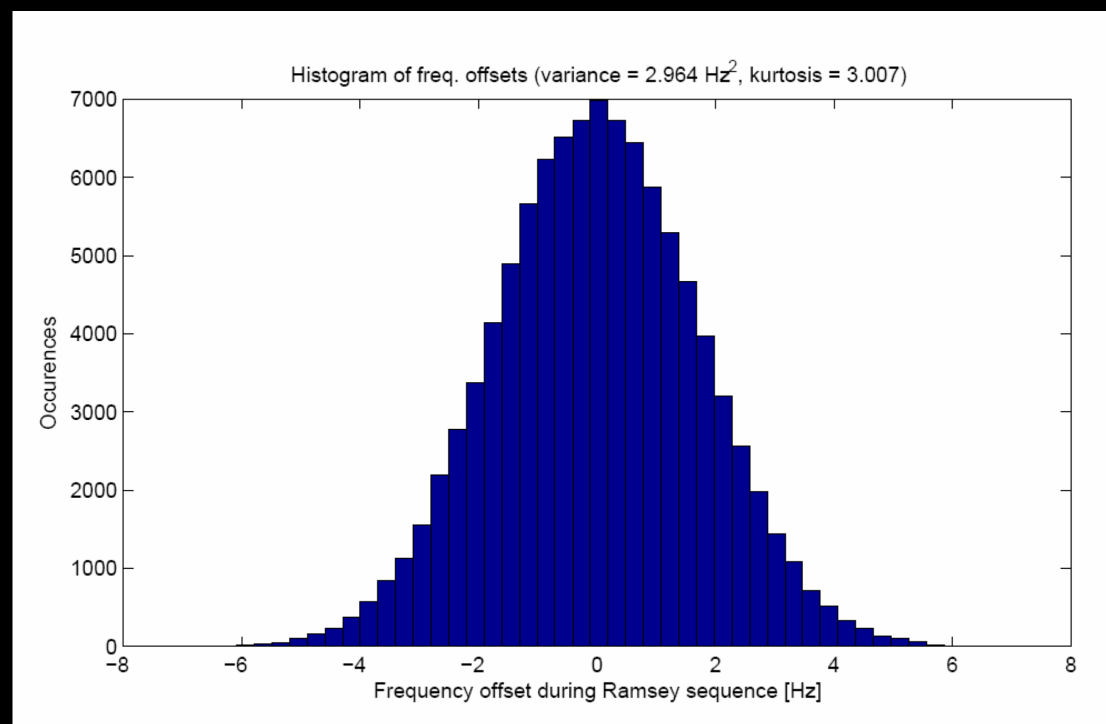
- This acts as our black box/oracle operation.

Goal: Phase Estimation

Frequency Priors and the Clock Oracle

Given $f - f_0$ we know that: $|k\rangle \to e^{-ik(f-f_0)T}|k\rangle$

We don't know $f - f_0$, but we can construct a prior probability distribution over $f - f_0$. Likewise, before, we didn't know which oracle our algorithm would be given, so we assigned each a probability.



Clock SDP

$$\rho^{Q} = \sum_{k,l} a_k |k\rangle\langle l|$$

$$\rho^{O} = \sum_{x,y} \sqrt{p_x}\sqrt{p_y}|x\rangle\langle y|$$

$$\Omega = \sum_{x} |x\rangle\langle x| e^{-i\Delta f_x \sigma_z T/2}$$

Goals:
1. Determine optimal initial state of Q
2. Determine optimal measurement

After one query

$$\Omega^{\dagger}\rho^{OQ}\Omega \rightarrow \sum_{x,y}\sum_{k,l} a_{x,y,k,l}|x\rangle\langle y| \otimes e^{i\Delta f_x kT}|k\rangle\langle l| e^{-i\Delta f_y lT}$$

Clock Cost Function

- Measure with a discrete, complete POVM $\{P_i^Q\}_i$. whose elements each correspond to a frequency guess, Δf_i
- Remote State Preparation:

$$\sigma_i = tr_Q(P_i^Q \rho^{OQ})$$

$$(\sigma_i)_{x,x} = P(measured\ \Delta f_i \cap frequency\ =\ \Delta f_x)$$

- Penalize guesses far from the true frequency

$$\langle \Delta f_x | A_i | \Delta f_x \rangle = (\Delta f_i - \Delta f_x)^2$$

- From the final density matrix and the remotely prepared mixed states, we can reconstruct this optimal POVM

Discussion

- Prior work has assumed a uniform probability distribution over clock oracles. *[4]*
- This technique is subject to discretization error
- Working on bounds – error appears small
- Al^+ Clock at NIST is a natural application of this technique. Uses only 1-2 ions and is the most accurate clock in the world (8.6×10^{-18}) *[5,9]*

Results

- Optimal initial states have equal amplitudes for *k* ions in the excited state and *N* – *k* in the ground state. e.g. for *N* = *3*:

$$|\psi_0\rangle = a_0\left(|0\rangle + |3\rangle\right) + a_1\left(|1\rangle + |2\rangle\right)$$

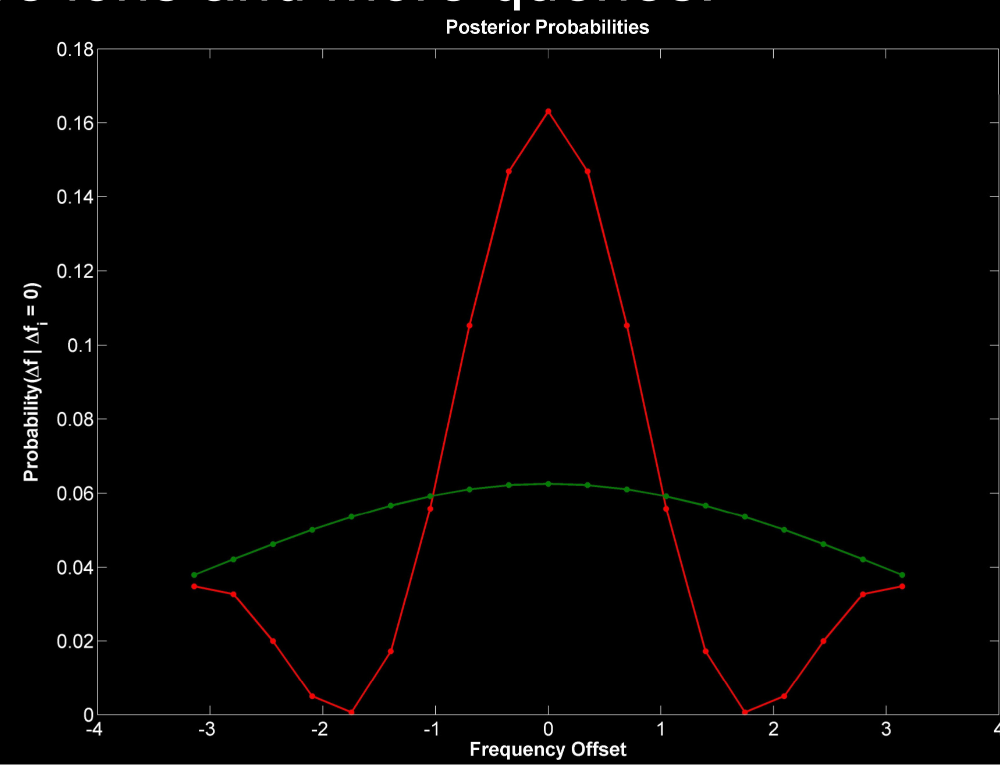
- One query on 2 ions is equivalent to 2 queries on one ion. Not necessarily true with more ions and more queries.

A narrower posterior corresponds to an increase in knowledge.

Green: Priors

Red: Posteriors

$P(\Delta f | \Delta f_i = 0)$



Conclusions

- This SDP formulation is a powerful and highly general way of describing quantum algorithms and quantum processes
- The size of the SDP grows quickly, but this technique remains applicable to many interesting problems.
- Here we analyzed quantum computers subject to decoherence, and developed a technique to examine the “quantumness” of a device.
- The application of our SDP to quantum clocks indicates that it likely has many other applications

References

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Algorithm Reconstruction

- SDP outputs a density matrix corresponding to the state of the computer at every timestep

- Purify the following pairs of states into A, where |A| = |O||Q|E|

$$D^{\dagger QE_t}\Omega^{\dagger QO}\rho^{OQE}(t)\Omega^{QO}D^{QE_t} \rightarrow \rho'^{OQEA}(t)$$

$$\rho^{OQE}(t+1) \rightarrow \rho^{OQEA}(t+1)$$

- Since we know that

$$\rho^{OQEA}(t+1) = U(t)^{\dagger QA}\rho'^{OQEA}(t)U(t)^{QA}$$

we can solve for the inter-query unitaries. Since the SDP yields the solution with the optimal probability of success, this reconstructs an optimal algorithm. (quantum or classical)

Measuring Quantum Resources

- Our SDP formulation will yield the algorithm with the optimal probability of success.
- But can we find an "alternate" algorithm with the same probability of success that uses fewer quantum resources?
- Quantify quantum resources via Von Neumann entropy

$$S(\rho) \equiv tr(\rho \; lg(\rho))$$

- By Schumacher's quantum coding, we know this is the minimum number of qubits needed to represent a quantum state *[10]*

Separate Computational Paths

- Let the optimal algorithm run normally until time t_s, then measure
- Quantify the quantum resources needed to reach timestep $t_s + 1$
  - Try to express $\rho^O(t_s)$ as the incoherent sum of $\rho_1^O(t_s),\ \rho_2^O(t_s) \ldots \rho_n^O(t_s)$
  - These independent computational paths can be classically combined.
  - Calculate entropy as:

$$S_T = tr(\rho_1^O) S\left(\frac{\rho_1^O}{tr(\rho_1^O)}\right) + tr(\rho_2^O) S\left(\frac{\rho_2^O}{tr(\rho_2^O)}\right) + \ldots$$

  - Since the full density matrix is pure: $S(\rho^O) = S(\rho^{QA})$

Splitting SDP

- Add the following constraints to a new SDP

$$\rho^{O}_{opt}(t) = \rho^{O}_{1}(t) + \rho^{O}_{2}(t)$$

$$tr_Q(\rho^{OQ}_{1}(t)) = \rho^{O}_{1}(t) \qquad t \geq t_s$$

$$tr_Q(\rho^{OQ}_{2}(t)) = \rho^{O}_{2}(t)$$

with the usual sets of constraints on these new density matrices. Here, the oracle density matrix is the optimal solution to our original SDP.

- Add an objective which maximizes the distance between the two subparts of the oracle density matrix.
- This will be a nonlinear objective so we use an iterative strategy

Splitting Results

- Recursively perform this splitting, calculating the entropy at each iteration.

8 Element Search/OR:

0. 2.056
2. 2.0096
4. 2.0093
8. 2.0093

4 Element PARITY

0. 2
2. 1
4. 0
8. 0

- Parity looks classical in the Hadamard basis.
- Entropy is a basis independent quantity.