# Closed timelike curves in measurement-based quantum computation

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# Outline

- Closed time-like curves (CTCs) = time travel
- Deutsch's model for CTCs
- How CTCs show up in measurement-based quantum computation
- CTC model by Bennett/Schumacher/Svetlichny
- Conclusion

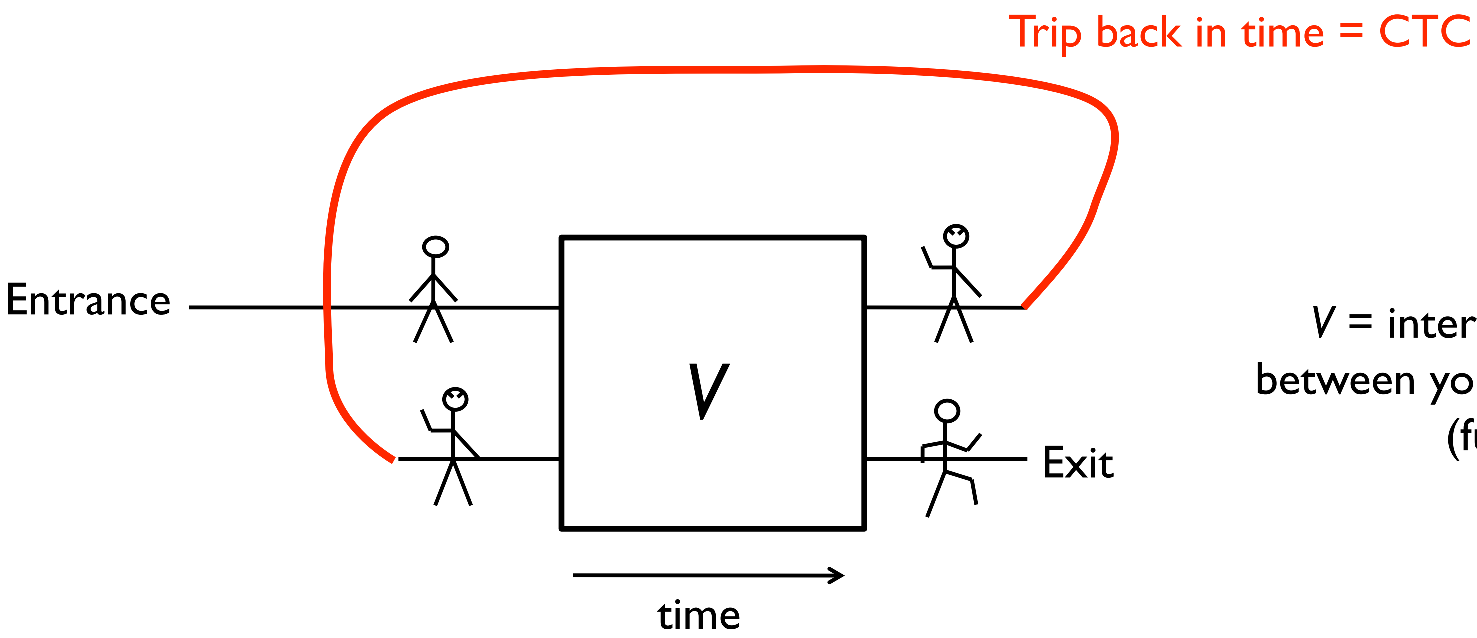
# Time travel

- To the future? Easy, use relativity.

- To the past? More involved:
  - Relativity predicts solutions with *closed timelike curves* (CTCs)= travel to the past
  - It's not known whether CTCs are physically possible, perhaps with clever black-hole engineering.
  - To avoid paradoxes, self-consistency conditions on time-travellers must apply – more on that later.

# Time travel - CTCs

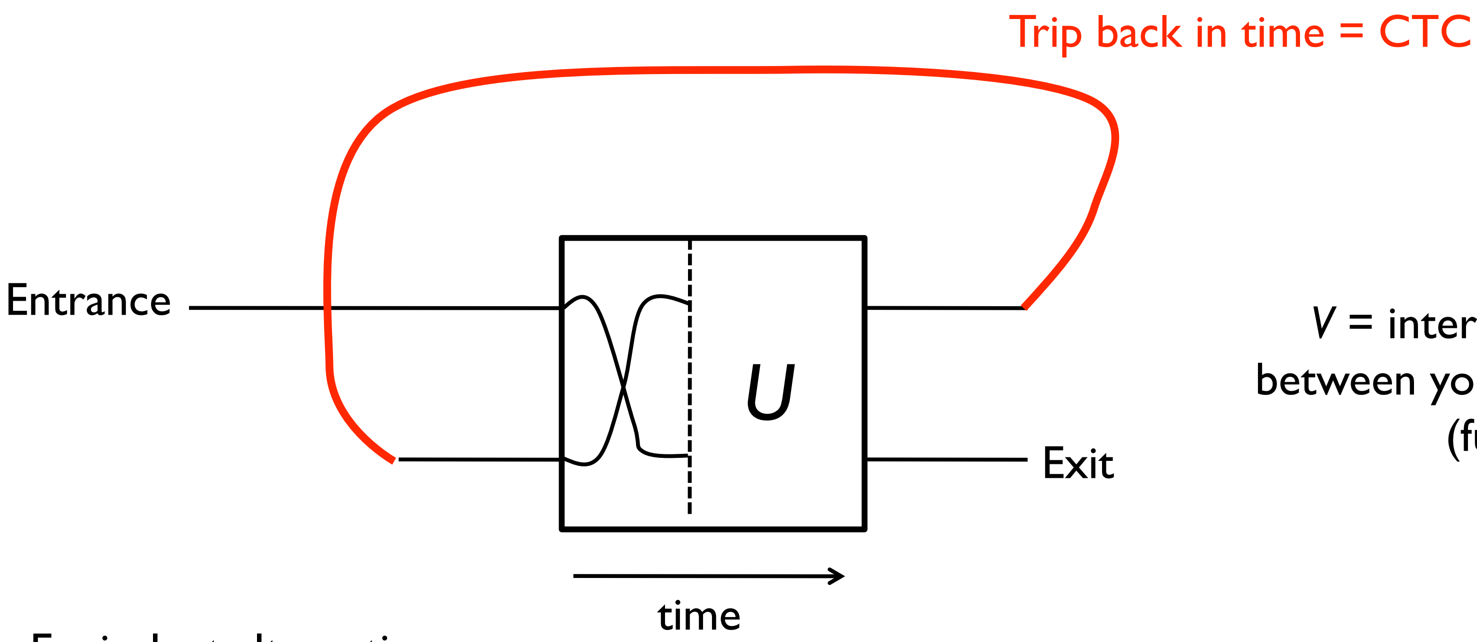
- Time travel scenario:




*V* = interaction region between you (past) and you (future)

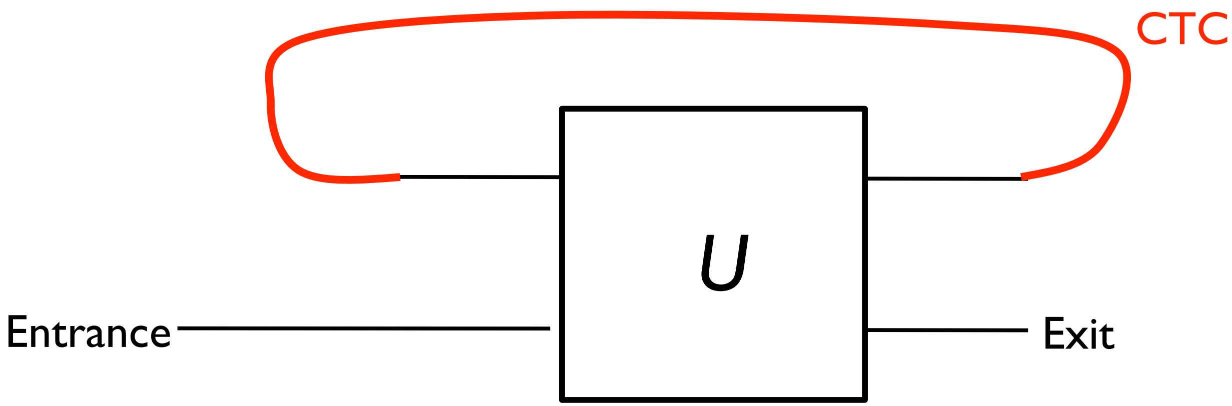
# Time travel - CTCs

- Time travel scenario:




*V* = interaction region between you (past) and you (future)

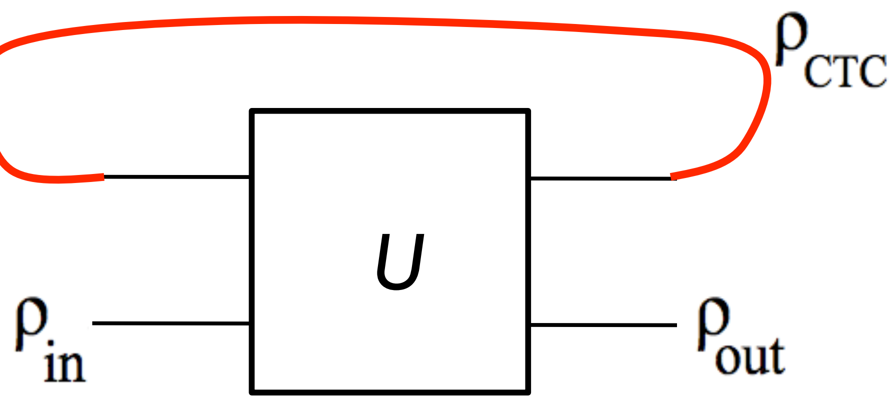
- Equivalent alternative:




*U* = interaction region

# Deutsch's model for CTCs

Deutsch, Phys. Rev. D 44, 3197 (1991)



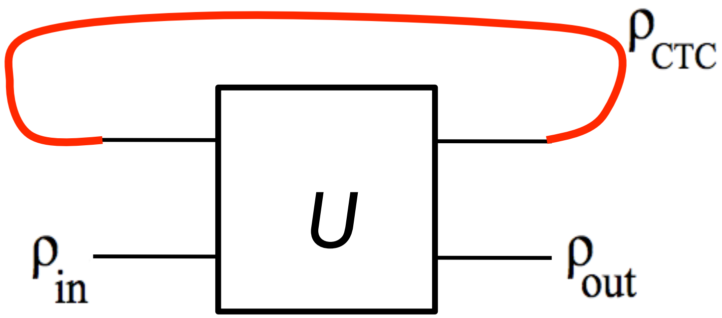

- $U$ = 2-qubit unitary
  - 1 qubit travels back in time
  - 1 qubit doesn't
- Initial state: $\rho_{CTC} \otimes \rho_{in}$
- After $U$: $U(\rho_{CTC} \otimes \rho_{in})U^{+}$
- Self-consistency condition: $\rho_{CTC} = Tr_B\left[U(\rho_{CTC} \otimes \rho_{in})U^{+}\right]$
- Deutsch showed that:
  - there's always at least 1 self-consistent solution;
  - there can be multiple such solutions;
  - each solution corresponds to an input-output map, in general non-linear.

# Deutsch's model for CTCs

Deutsch, Phys. Rev. D 44, 3197 (1991)

- Some characteristics:

  - avoids paradoxes;
  - Under-determination of multiple solutions/maps: maxent? wormhole initial conditions? …

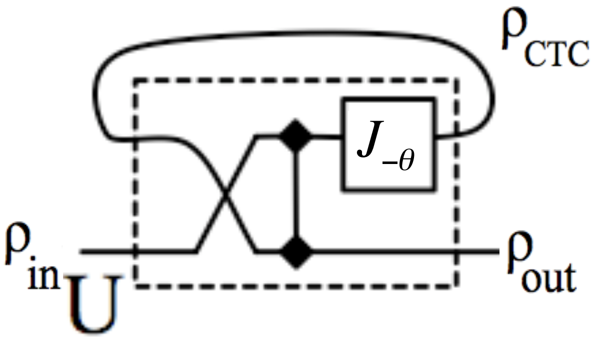


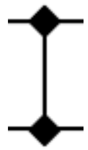

- **Computational power** of Deutsch's CTCs :

  Non-orthogonal state discrimination? Solution to NP-complete problems?

  - Bacon arXiv:quant-ph/0309189v3 (solves NP-complete problems)
  - Brun *et al.* arXiv:0811.1209v2 (non-orthogonal state discrimination)
  - Aaronson, Watrous: arXiv:0808.2669v1 (CTCs -> PSPACE)
  - Bennett *et al.* arXiv:0908.3023v2 (criticism to results above)
  - Cavalcanti, Menicucci arXiv:1004.1219v2 (criticism of the criticism…)

# Deutsch - example



$$= CTR - Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad J_{-\theta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{-i\theta} \\ 1 & -e^{-i\theta} \end{pmatrix}$$

$$\rho_{in} = \frac{1}{2}(1 + \vec{n} \cdot \vec{\sigma}) \qquad \rho_{CTC} = \frac{1}{2}(1 + \vec{m} \cdot \vec{\sigma}) \qquad \rho_{out} = \frac{1}{2}(1 + \vec{r} \cdot \vec{\sigma})$$

- Self-consistency:

$$\begin{aligned} m_x &= n_z, \\ m_y &= m_z(n_x \sin\theta - n_y \cos\theta), \\ m_z &= m_z(n_x \cos\theta + n_y \sin\theta). \end{aligned}$$

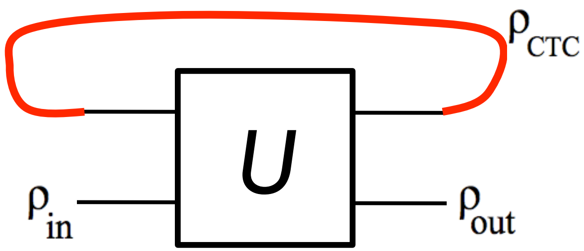
- Solution for generic input:

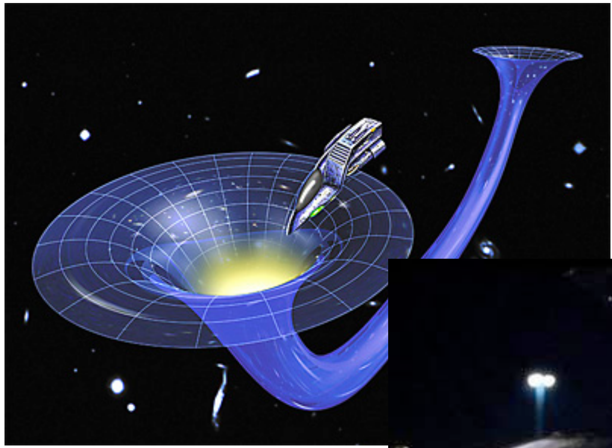
$$\begin{cases} \rho_{CTC} : \vec{m} = (n_z, 0, 0) \\ \rho_{out} : \vec{r} = (n_z^2, 0, 0) \end{cases}$$

Non-linear map!

- Alternative solution for $|\psi_{in}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\theta}|1\rangle\right) : \rho_{CTC} = \rho_{out} : \vec{m} = (0, 0, m_z)$

# Testing Deutsch's model



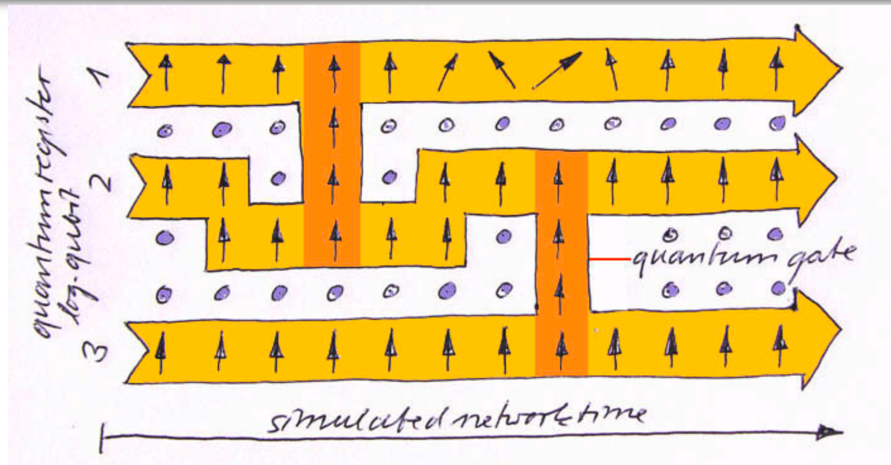



- Discussion of computational power of CTCs uses Deutsch's model. In the absence of experiments, how to check if the model is sound?

- We'll see that **measurement-based quantum computation** offers an answer.

# The one-way model of quantum computing

- Proposed by Raussendorf/Briegel
  [PRL 86, 5188 (2001)]



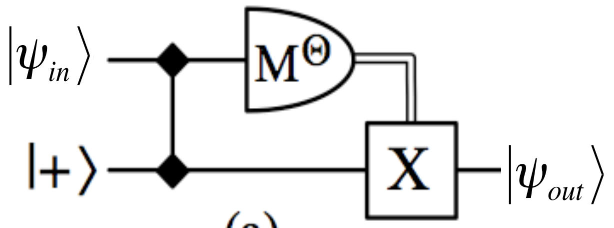
Credit: Robert Raussendorf

- Consists in:
  - Preparation of standard entangled states via Heisenberg interactions – *cluster* states;
  - Adaptive sequence of 1-qubit measurements.

- Computational resource = quantum correlations of initial state

- Algorithm = choice of adaptive sequence of measurements

# Example: *J* gate



(a)

$$|\psi_{in}\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |+\rangle \equiv 1/\sqrt{2}\left(|0\rangle + |1\rangle\right)$$

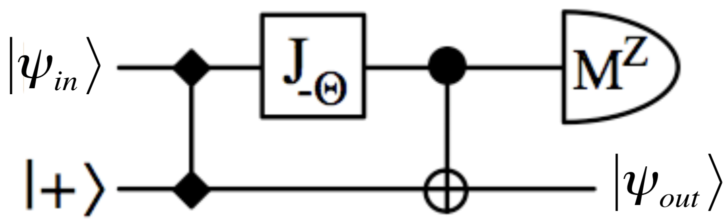
$$M^{\Theta} = \text{ Meas. on basis } \begin{Bmatrix} |0\rangle + e^{i\theta}|1\rangle \leftrightarrow \text{ outcome } s1 = 0 \\ |0\rangle - e^{i\theta}|1\rangle \leftrightarrow \text{ outcome } s1 = 1 \end{Bmatrix}$$

- Simple calculation shows that $|\psi_{out}\rangle = J_{-\theta}|\psi_{in}\rangle$

- Circuit can be represented as command sequence:

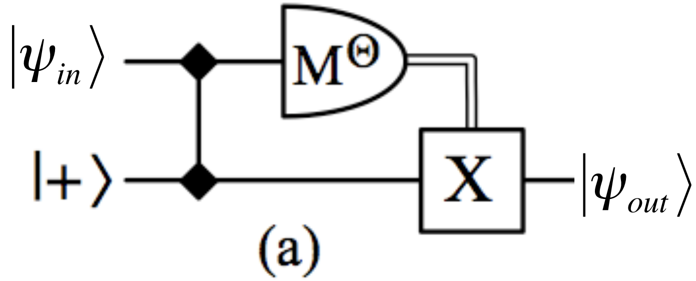
$$X_2^{s1} M_1^{\theta} |G\rangle, \qquad |G\rangle \equiv CTR_Z |\psi_{in}\rangle_1 \otimes |+\rangle_2$$

- Equivalent circuit – measure *Z*, implement *CTR-X* (*CNOT*) coherently:



(b)

# *J* gate in MBQC = CTC




$$\Leftrightarrow \quad X_2^{s1} M_1^{\theta} |G\rangle, \quad |G\rangle \equiv CTR_Z |\psi_{in}\rangle_1 \otimes |+\rangle_2$$

- Stabilizers of state $|G\rangle$ :

$$\begin{cases} (Z_1 \otimes X_2)^0 = Z_1^0 \otimes X_2^0 = 1 \text{ (identity)} \\ (Z_1 \otimes X_2)^1 = Z_1^1 \otimes X_2^1 \end{cases}$$

That is, $Z_1^{s1} \otimes X_2^{s1}$ is stabilizer independently of the outcome $s_1$ of the measurement on qubit 1:

$$Z_1^{s1} \otimes X_2^{s1} |G\rangle = |G\rangle$$

- We can then perform stabilizer manipulation:

$$X_2^{s1} M_1^{\theta} |G\rangle = X_2^{s1} M_1^{\theta} (Z_1^{s1} X_2^{s1} |G\rangle)$$

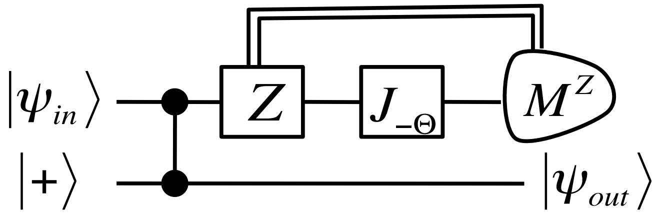
$$= X_2^{s1+s1} M_1^{\theta} Z_1^{s1} |G\rangle = \boxed{M_1^{\theta} Z_1^{s1} |G\rangle}$$

**Time-travel** situation: we need to apply *Z* depending on outcome of measurement not yet made.

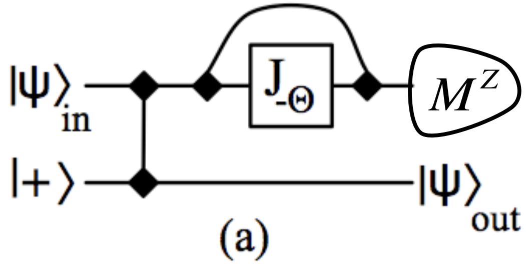
# *J* gate in MBQC = CTC

- Putting this CTC in Deutsch format:

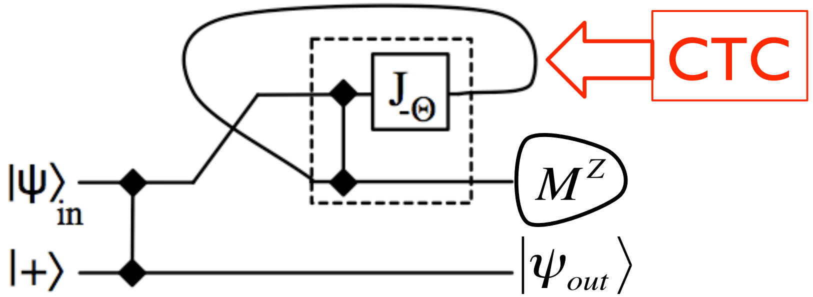
$$M_1^{\theta} Z_1^{s1} |G\rangle \Leftrightarrow$$



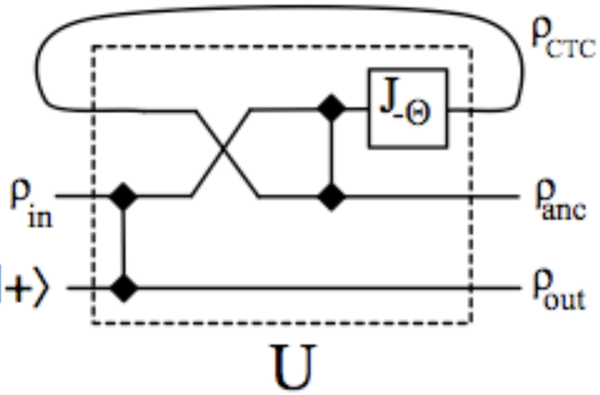

$\Leftrightarrow$



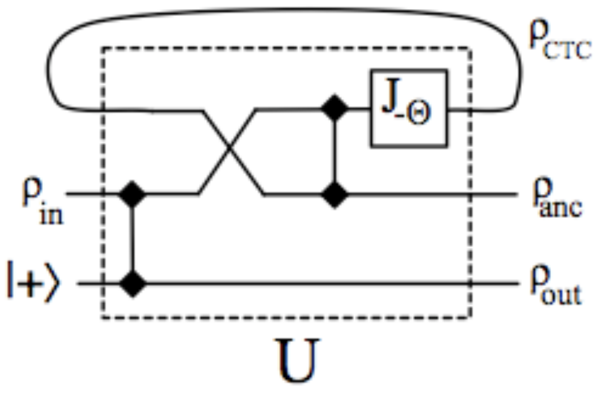

$\Leftrightarrow$




$\Leftrightarrow$



# Comparing the one-way model with Deutsch:

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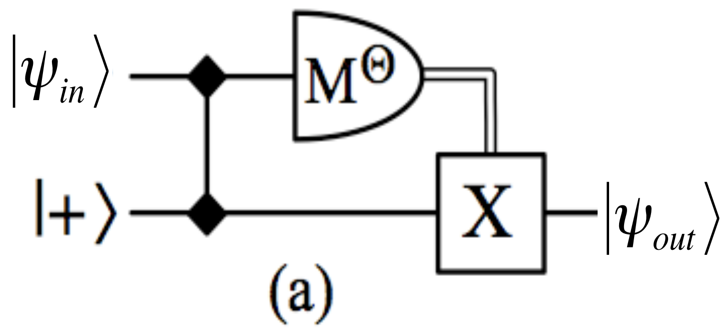



Deutsch:

$$\begin{cases} \rho_{in} : \vec{n} = (n_x, n_y, n_z) \\ \rho_{anc} : \vec{m} = (n_z^2, 0, 0) \\ \rho_{out} = \rho_{CTC} : \vec{r} = (n_z, 0, 0) \end{cases}$$

$$\rho_{in} = \frac{1}{2}\begin{pmatrix} 1+n_z & n_x - in_y \\ n_x + in_y & 1-n_z \end{pmatrix} \longrightarrow \rho_{out} = \frac{1}{2}\begin{pmatrix} 1 & n_z \\ n_z & 1 \end{pmatrix}$$

One-way model:

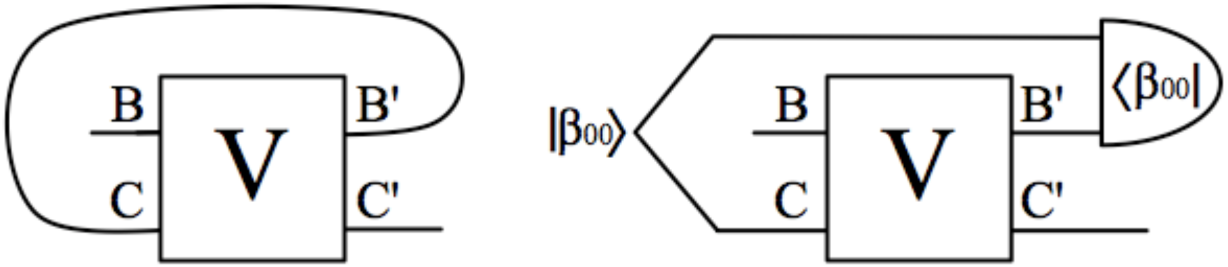
$$|\psi_{in}\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi_{out}\rangle = J_{-\theta}|\psi_{in}\rangle = \alpha|+\rangle + \beta e^{-i\theta}|-\rangle$$

Deutsch's prediction is *different* from the one based on the one-way model. What's wrong with Deutsch?

# CTCs: model by Bennett/Schumacher/Svetlichny (BSS)

- Bennett and Schumacher, unpublished (2002) – see seminar http://bit.ly/crs8Lb
- Rediscovered independently by Svetlichny (2009) - arXiv:0902.4898v1
  - Related work on black holes by Horowitz/Maldacena (2004), Preskill/Gottesman (2004)




$$|\beta_{00}\rangle \equiv \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

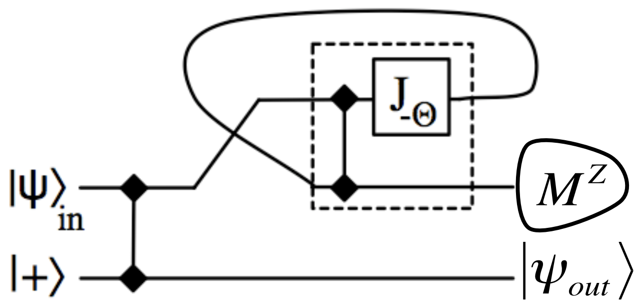
CTC

Simulation using teleportation and post-selection: B'=C

- We post-select projections onto $|\beta_{00}\rangle$
  - Postselection successful: state B' is teleported back in time (state C = state B')
  - Simulation works only when post-selection happens -> finite probability of success.

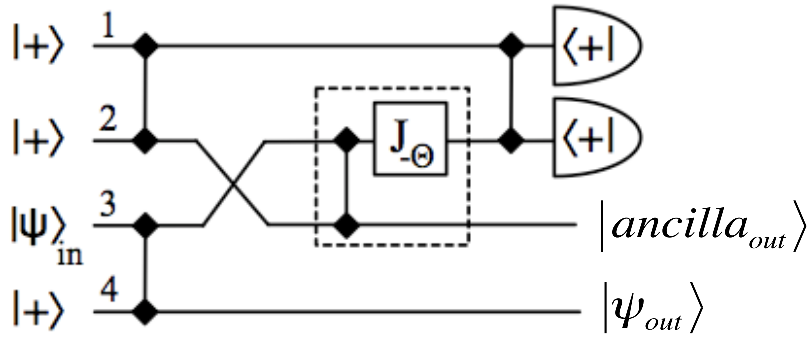
What are BSS's predictions for our CTC?

# BSS x MBQC




CTC we studied

$\Longrightarrow$




Its BSS simulation circuit

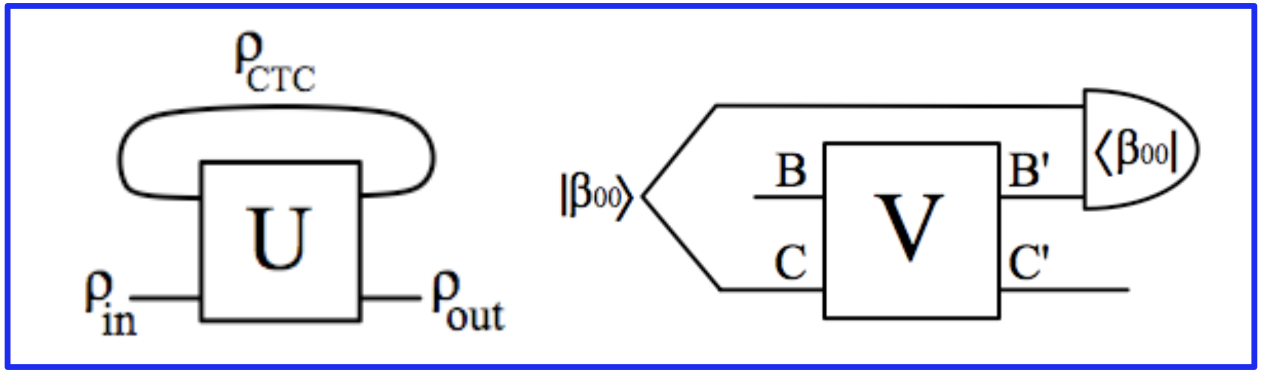
- Simple calculation shows that BSS circuit implements (probabilistically) the map

$$|\psi_{in}\rangle \rightarrow |\psi_{out}\rangle = J_{-\theta}|\psi_{in}\rangle$$

… recovering exactly MBQC's prediction!

BSS is the right model to explain CTCs in MBQC, and not Deutsch's…

# Deutsch/BSS comparison

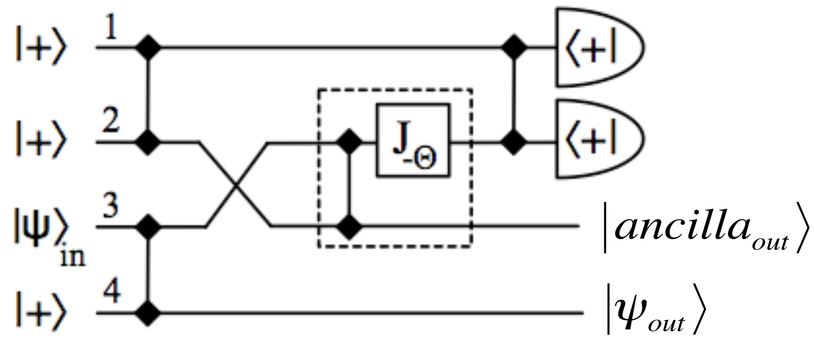



- Deutsch's self-consistency is achieved with artificial mixed-states, CTC qubit sent back in time decorrelated with other systems.
- BSS preserves entanglement and correlations of CTC qubit due to teleportation step.

- Deutsch solutions with pure-state CTC qubit coincide with BSS solution.

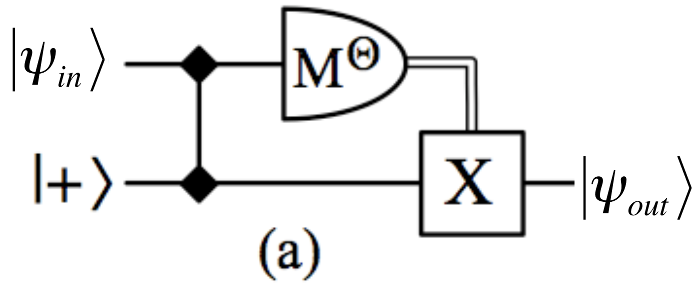
# MBQC as deterministic simulations of CTCs

- Stabilizer techniques enable us to simplify BSS circuits
  - Z deletion;
  - Local complementation.

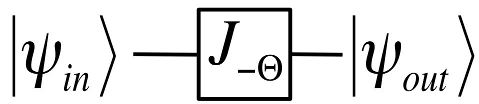



- Some BSS circuits reduce to MBQC patterns that deterministically simulate a unitary.
  - flow, gflow theorems.

- For example, the BSS circuit above is equivalent to:




- Or more simply:



# Conclusions

- CTCs appear in MBQC; we analyze them using two CTC models.
- BSS's model agrees with MBQC.
- Deutsch's model is in conflict with MBQC – the CTC qubit is sent to past stripped of its entanglement.
- We characterize a class of CTCs that admit deterministic simulation circuits using the BSS model.
- More work is needed to better understand implications of the BSS model:
  - MQ + Deutsch's CTCs = PSPACE (Aaronson/Watrous 2008)
  - BSS is associated with complexity class PostBQP=PP (Aaronson 2004)
  - See recent work (and experiment) by Lloyd *et al.* : arXiv:1005.2219v1

# PP versus PSPACE

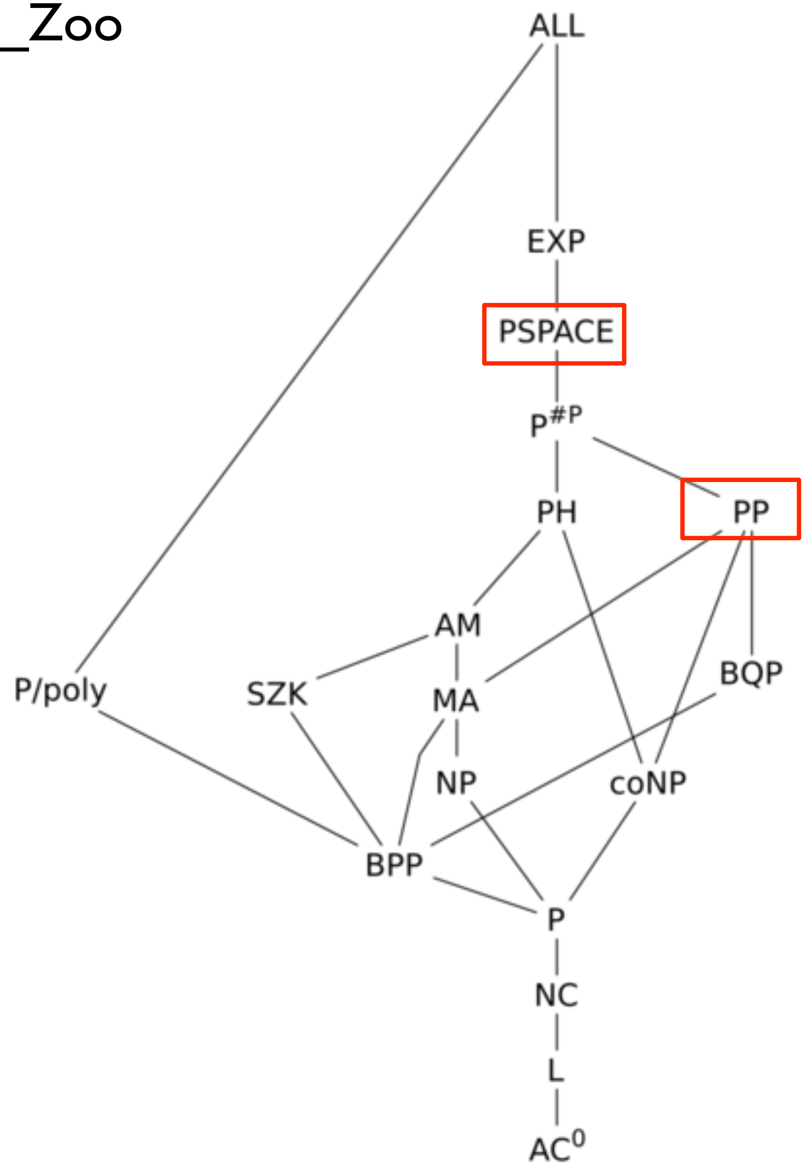
From http://qwiki.stanford.edu/wiki/Complexity_Zoo

## PP

Like BPP, PP is a class defined in an attempt to find out what randomness allows us to do algorithmically. Formally, PP is the class of problems solvable by an NP machine such that, given a "yes" instance, strictly more than 1/2 of the computation paths accept, while given a "no" instance, strictly less than 1/2 of the computation paths accept

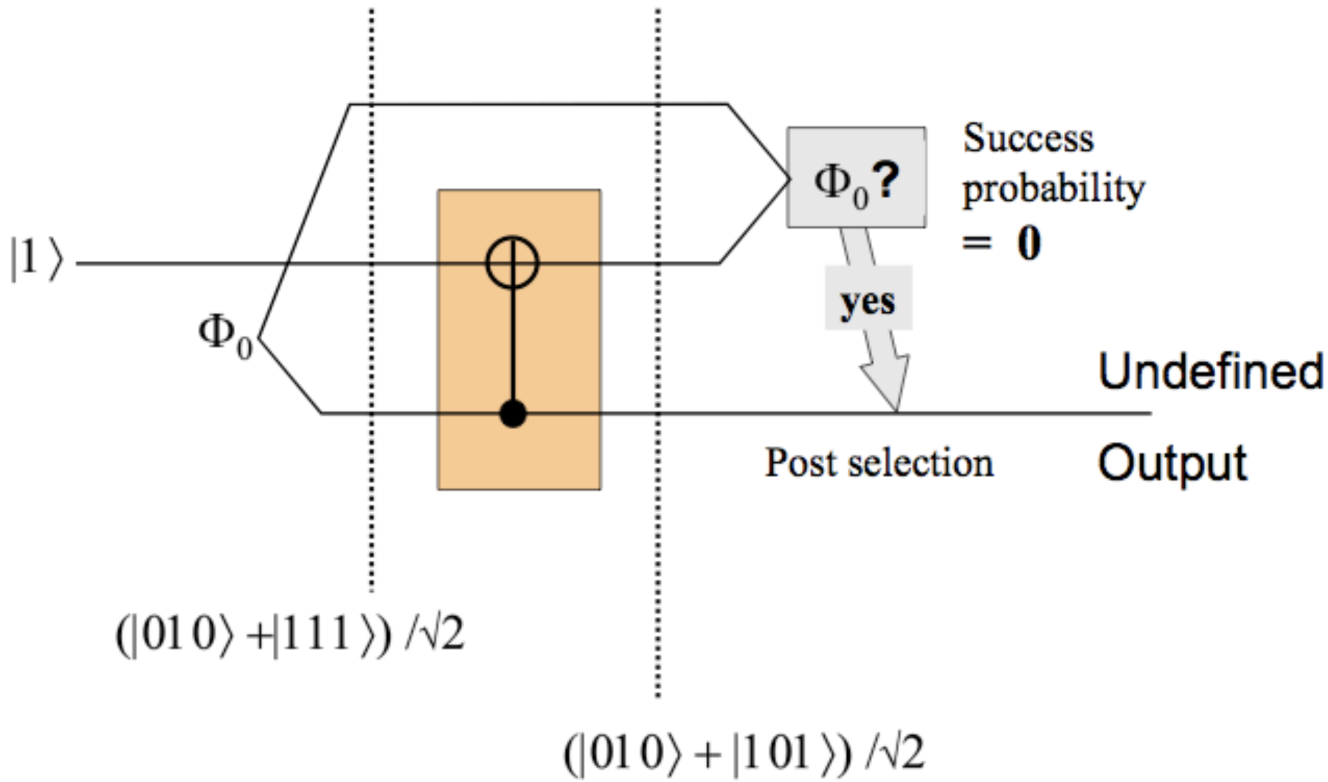
## PSPACE

Whereas P is a class of problems that can be solved in a polynomially-bounded amount of time, PSPACE is the class of problems that can be solved by a deterministic Turing machine that uses only a polynomially-bounded amount of space, regardless of how long the computation takes.



# How BSS deals with the grandfather paradox

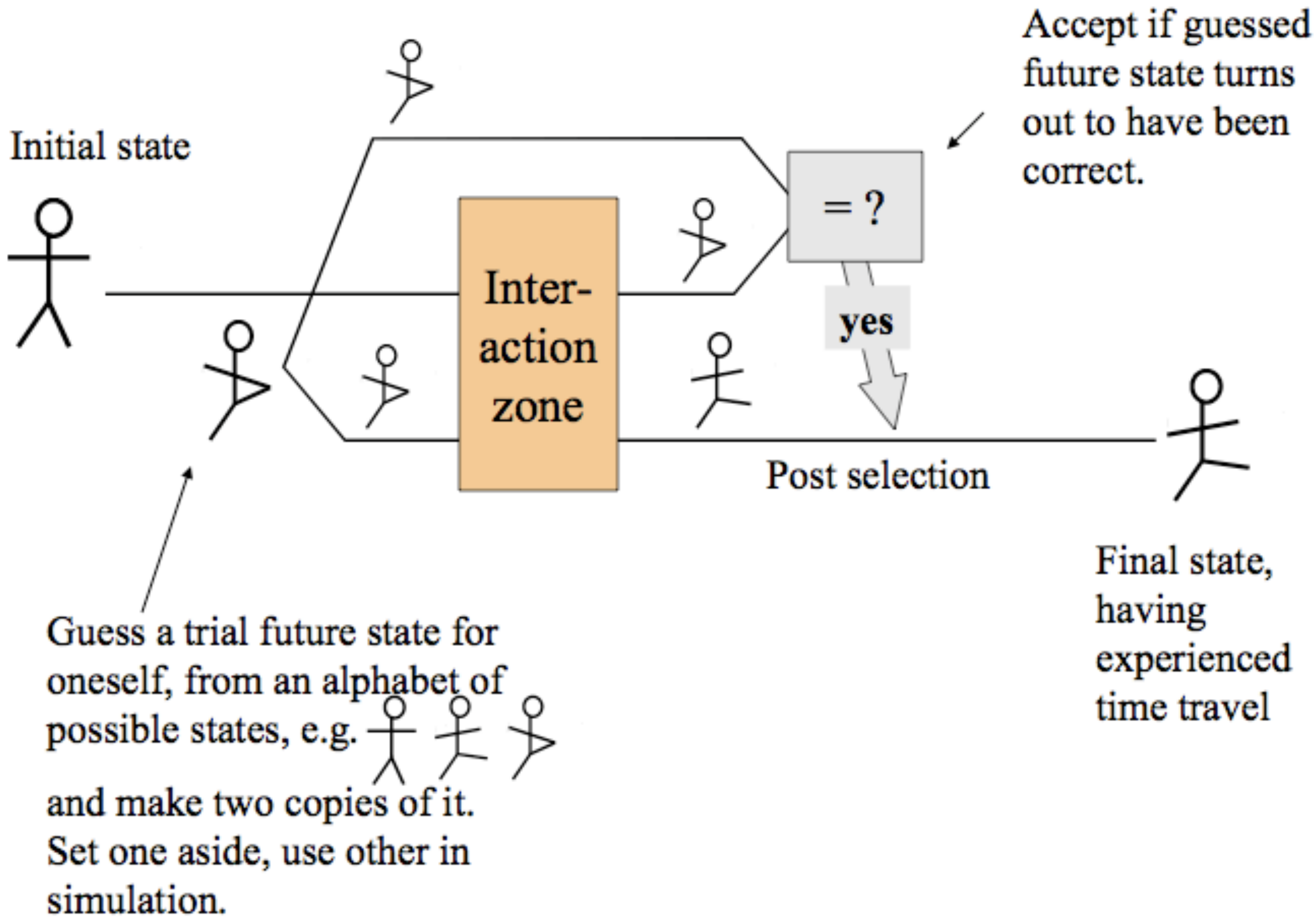
- From Bennett's talk slides: http://bit.ly/crs8Lb




- Paradoxical combinations of input and unitary result in post-selection with success probability *p=0*.

# Bennett’s classical motivation for the BSS model

Woody Allen MC can be used to simulate time travel without need of any exotic physical equipment.




From Bennett’s talk (2005).