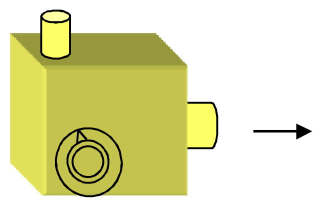
Back to realist approaches

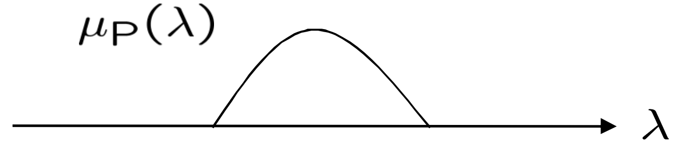
(this time allowing for hidden variables)

An ontological model of an operational theory

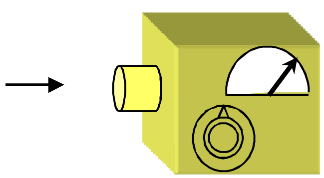
Preparation
P



$$\int \mu_{\mathrm{P}}(\lambda) d\lambda = 1$$

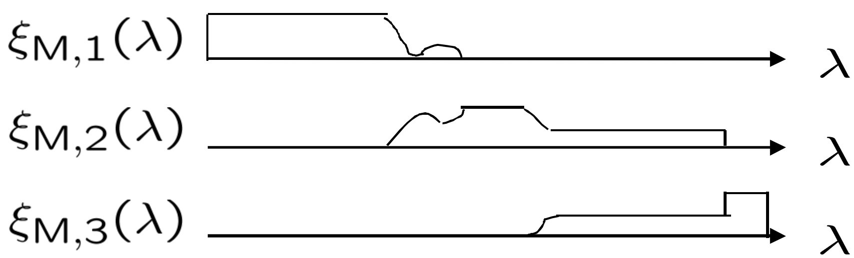


Measurement
M



$$0 \leq \xi_{\mathrm{M},k} \leq 1$$

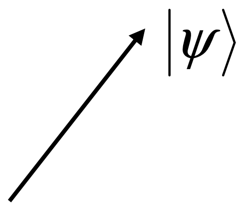
$$\sum_k \xi_{\mathrm{M},k}(\lambda) = 1 \text{ for all } \lambda$$

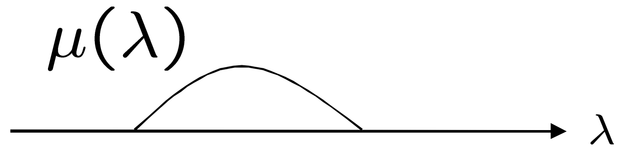


$$p(k|\mathrm{P},\mathrm{M}) = \int d\lambda \, \xi_{\mathrm{M},k}(\lambda) \, \mu_{\mathrm{P}}(\lambda)$$

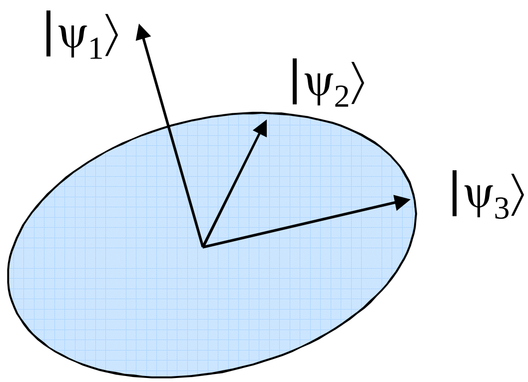
Deterministic hidden variable model for pure states and projective measurements

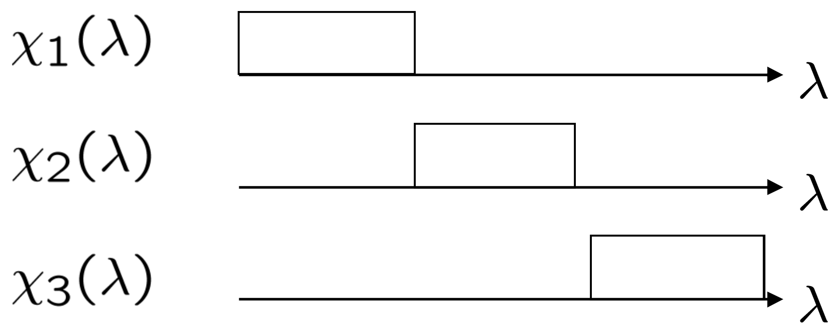
$\int \mu(\lambda) d\lambda = 1$





$\sum_k \chi_k(\lambda) = 1$ for all λ





It is assumed that the outcomes are deterministic given λ

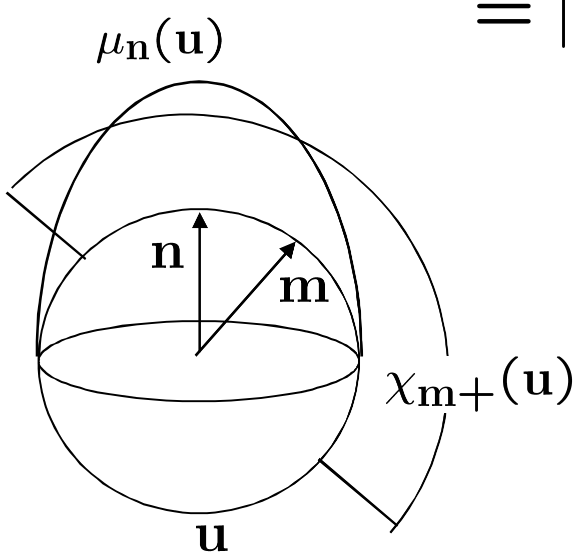
$$|\langle \psi | \psi_k \rangle|^2 = \int d\lambda \mu(\lambda) \chi_k(\lambda)$$

Example: the Kochen-Specker model for a 2d system

$$|+\mathbf{n}\rangle \leftrightarrow \qquad \mu_{\mathbf{n}}(\mathbf{u}) = \begin{cases} \frac{1}{\pi}\mathbf{n}\cdot\mathbf{u} & \text{for } \mathbf{n}\cdot\mathbf{u} > 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$|+\mathbf{m}\rangle \leftrightarrow \qquad \chi_{\mathbf{m}+}(\mathbf{u}) = \begin{cases} 1 & \text{for } \mathbf{m}\cdot\mathbf{u} > 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$\int \mu_{\mathbf{n}}(\mathbf{u})\chi_{\mathbf{m}+}(\mathbf{u})d\mathbf{u} = \frac{1}{2}(1+\mathbf{m}\cdot\mathbf{n}) = |\langle +\mathbf{m}| +\mathbf{n}\rangle|^2$$



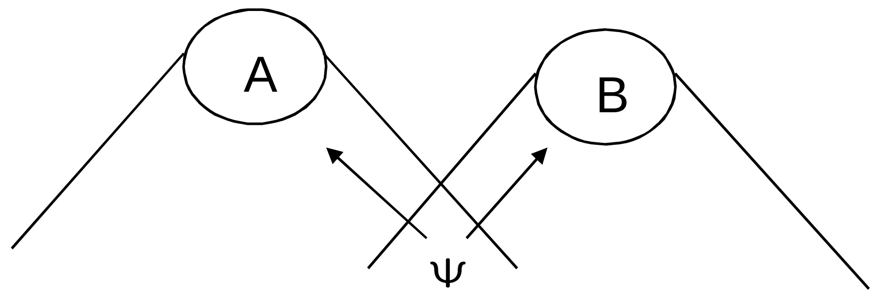
The KS model cannot be generalized to mixed states, POVMs or higher dimensions

Example: Statistically restricted classical theories

Consider Einstein's version of the EPR argument

Suppose A and B share

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |1\rangle|1\rangle\right)$$



If A measures $\{|0\rangle, |1\rangle\}$

B's state becomes $|0\rangle$ with probability 1/2
$|1\rangle$ with probability 1/2

If A measures $\{|+\rangle, |-\rangle\}$

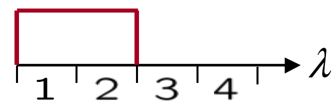
B's state becomes $|+\rangle$ with probability 1/2
$|-\rangle$ with probability 1/2

"Steering"

$$\mu(\lambda',\lambda)=\frac{1}{4}([11]+[22]+[33]+[44])$$

Alice's initial knowledge of B

If A measures {1,2} vs. {3,4}

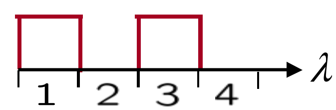


with prob. 1/2

Her knowledge of B is updated to

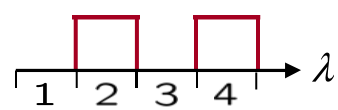
with prob. 1/2

If A measures {1,3} vs. {2,4}



with prob. 1/2

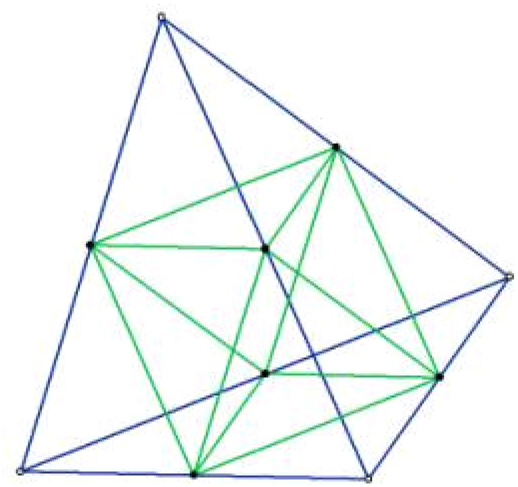
Her knowledge of B is updated to



with prob. 1/2

In a statistically restricted classical theory

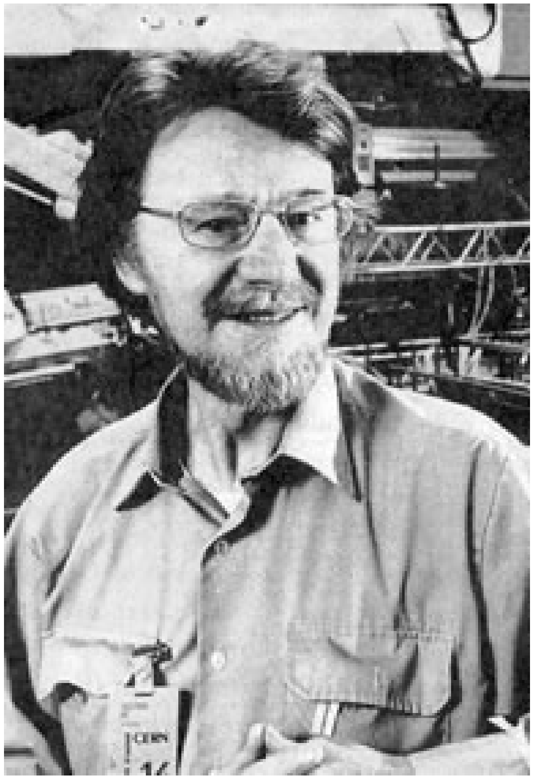
the convex set of operational states exhibits
- Convexly extremal states can be classically mixed
- non-simplicial shape / ambiguous mixtures
- Convexly extremal states can be correlated



Categorizing quantum phenomena

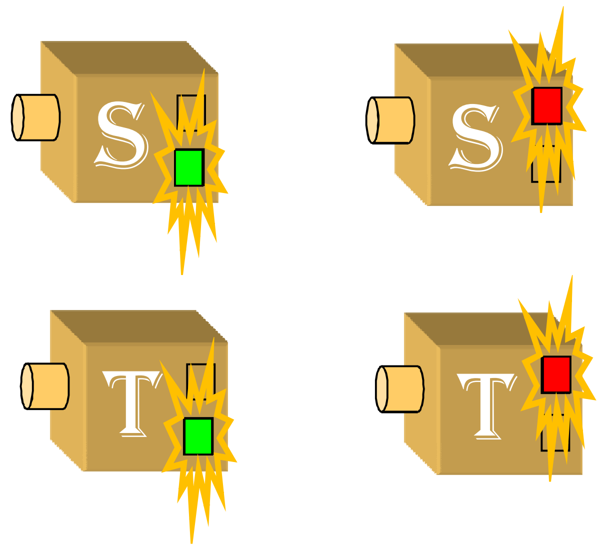
Those arising in a restricted statistical classical theory	Those not arising in a restricted statistical classical theory
Noncommutativity Entanglement Collapse Wave-particle duality Teleportation No cloning Key distribution Improvements in metrology Quantum eraser Coherent superposition Pre and post-selection “paradoxes” Others…	Bell inequality violations Contextuality Computational speed-up Certain aspects of items on the left Others…
Type 1 Nonclassicality	**Type 2 Nonclassicality**

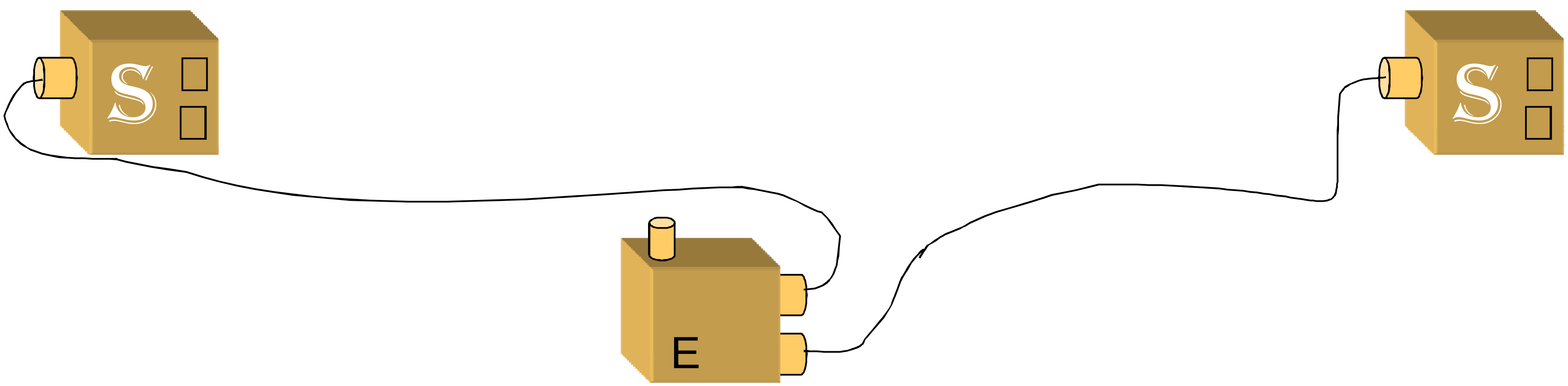
Bell's theorem

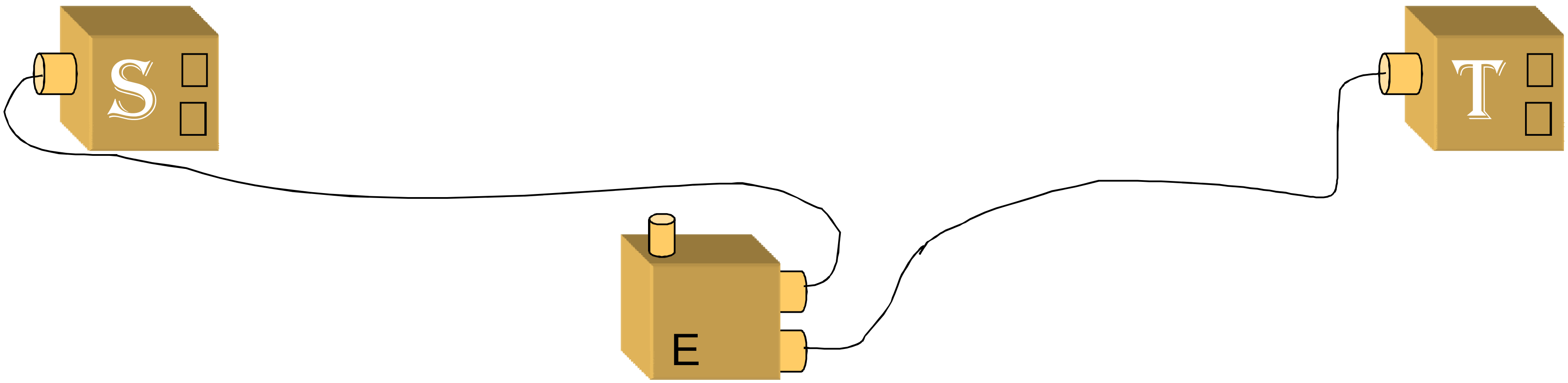


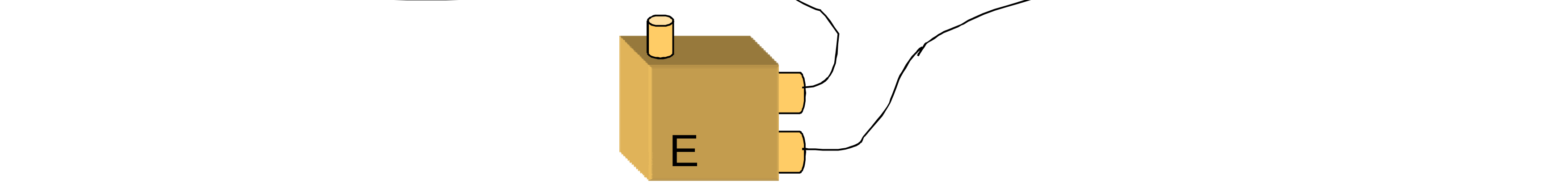
John S. Bell
(1928-1990)

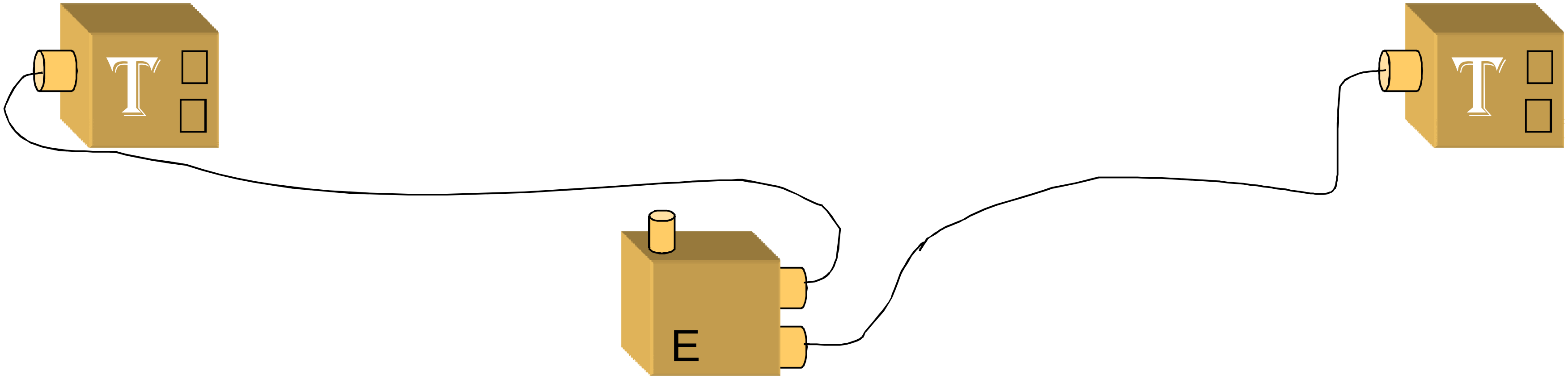
A pair of two-outcome measurements




S
S
E


S
E
T


T
S
E

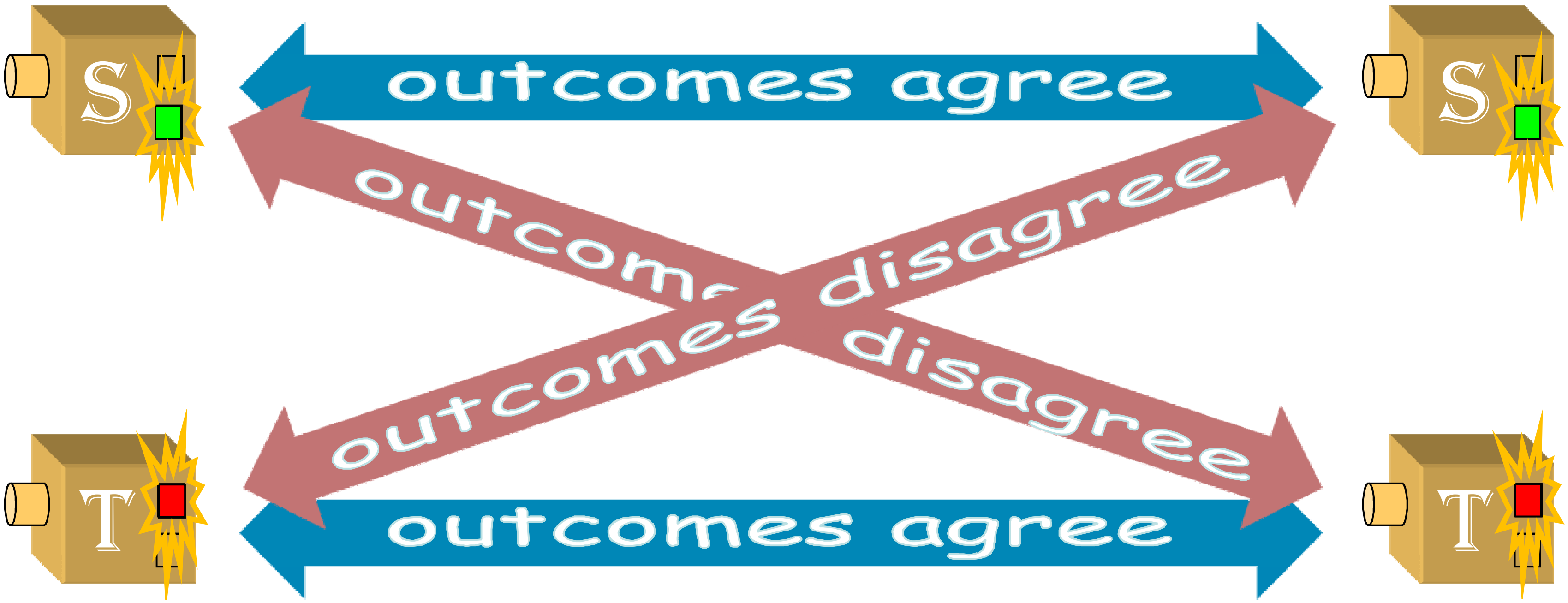

T
E
T

There are two possible measurements, S and T, with two outcomes each: green or red

Suppose which of S or T occurs at each wing is chosen at random

Scenario 1

1. Whenever the same measurement is made on A and B, the outcomes always agree	S and S or T and T
2. Whenever different measurements are made on A and B, the outcomes always disagree	S and T or T and S

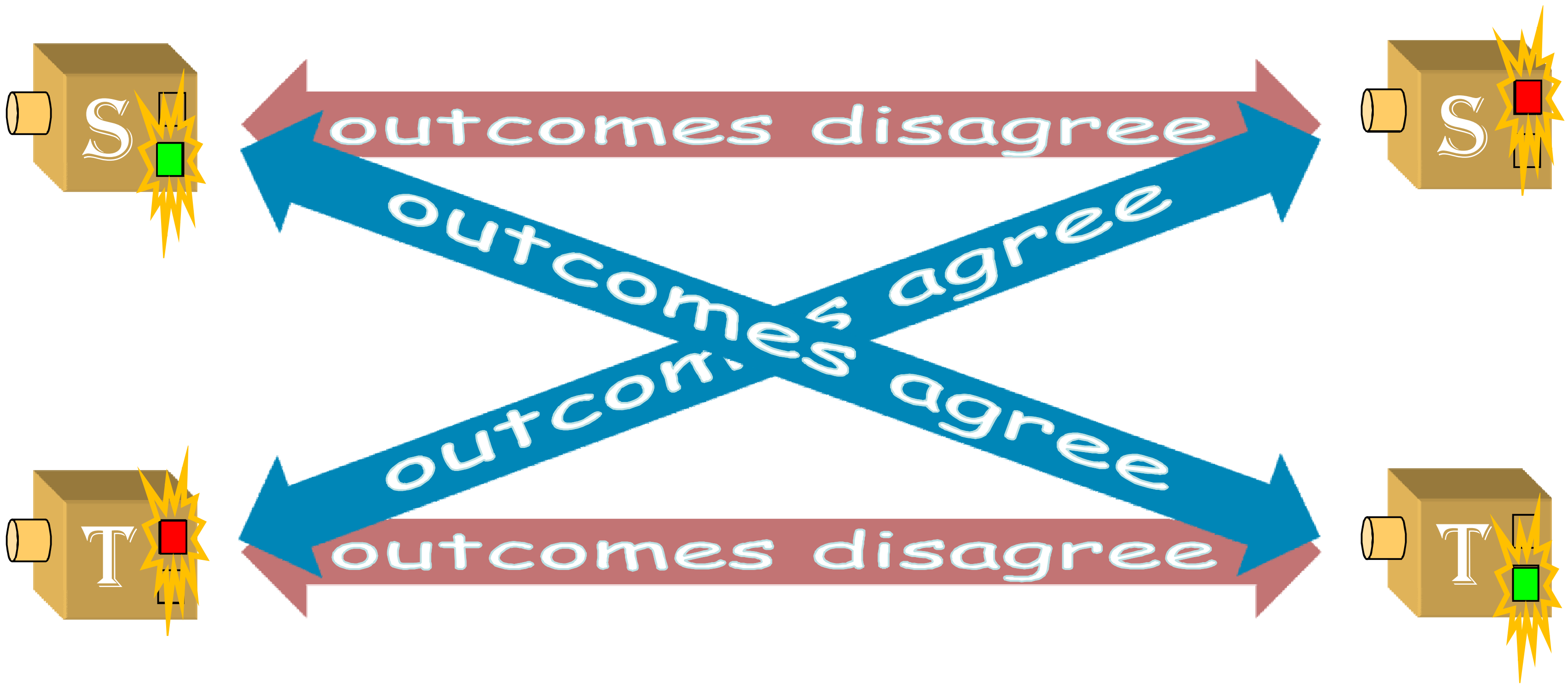

S
S
T
T
outcomes agree
outcomes agree
outcomes disagree
outcomes disagree

There are two possible measurements, S and T, with two outcomes each: green or red

Suppose which of S or T occurs at each wing is chosen at random

Scenario 2

1. Whenever the same measurement is made on A and B, the outcomes always disagree	S and S or T and T
2. Whenever different measurements are made on A and B, the outcomes always agree	S and T or T and S


S
S
T
T
outcomes disagree
outcomes agree
outcomes agree
outcomes disagree

There are two possible "measurements", S and T,
with two outcomes each: green or red

Suppose which of S or T occurs at each wing is chosen at random

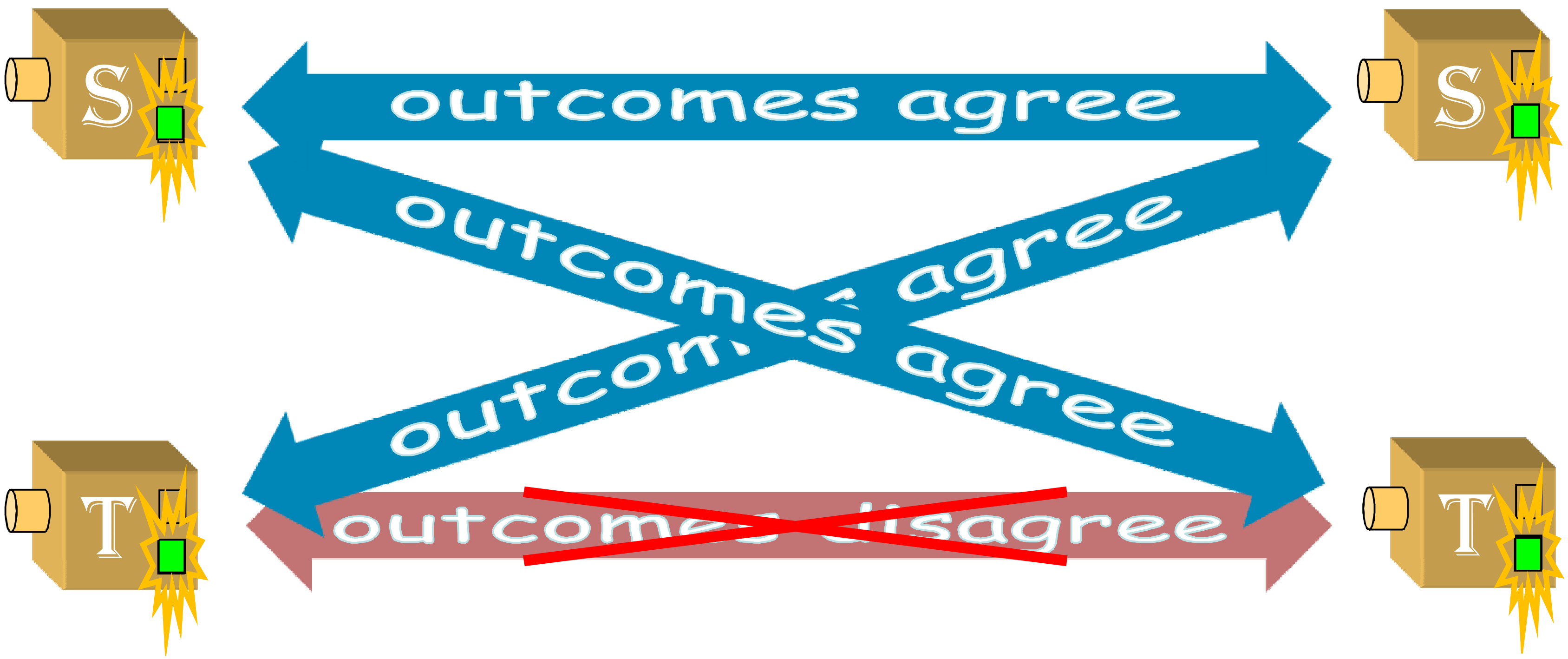
Scenario 3

1. Whenever the measurement T is made on both A and B, the outcomes always disagree

T and T

2. Otherwise, the outcomes always agree

S and S
or
S and T
or
T and S



The game can be won at most 75% of the time by local strategies

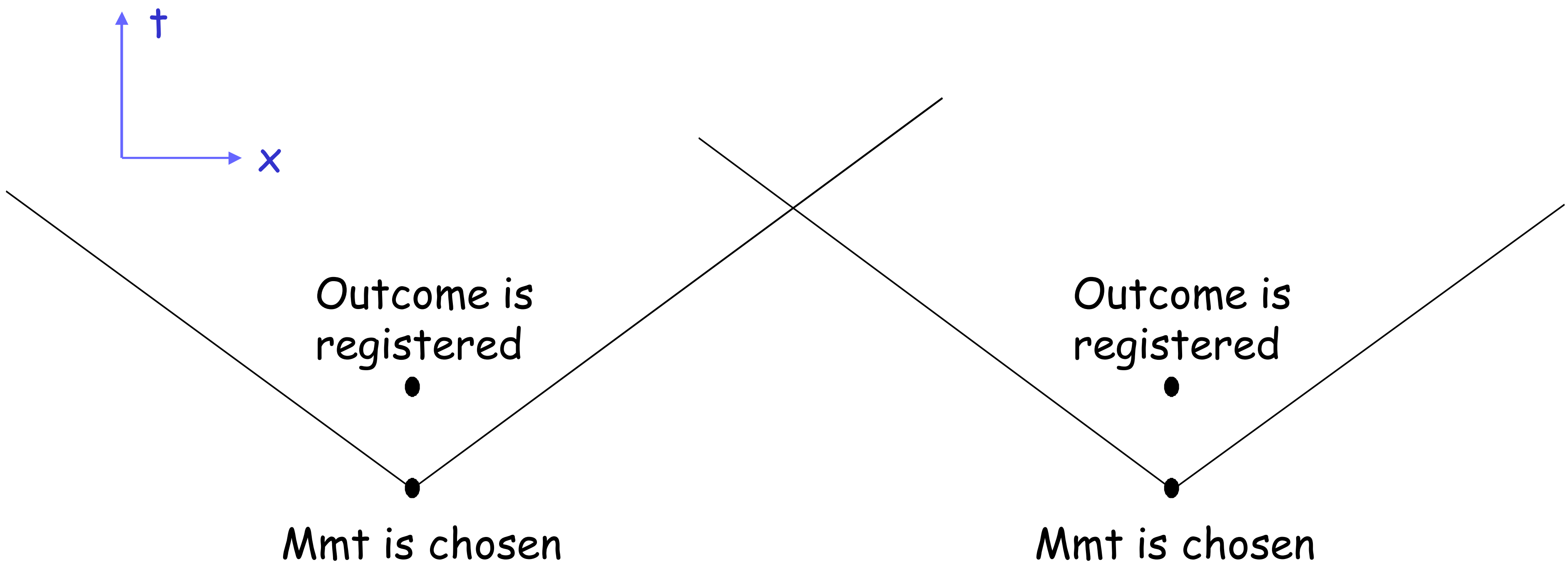
Using quantum theory, it can be won $\simeq$85% of the time

Q: How could you cheat and win the game all the time?

A: Communication of the choice of measurement in one wing to the system in the opposite wing

But there's a problem…

Tension with the theory of relativity



Experiment can distinguish:
1) the quantum predictions
2) the predictions of any locally causal theory

Quantum theory is corroborated!

Would access to randomness help to generate the correlations?

No. It will only decrease the degree of correlation

Is the proof robust to experimental imperfections? (e.g. the detector sometimes registers the wrong outcome)

Yes. The Bell inequality may still be violated.

If the detector inefficiencies are sufficiently high, can particles obeying local causality simulate the correlations on the detected pairs?

Yes. This is the detector loophole.

Is there a problem if the choice of measurement is made before the particles are sent to the detectors?

Yes. This is the locality loophole.

When seeking a realist explanation of these experiments, the mystery is the tension between:

1) No superluminal signalling (independence of statistics at one wing on choice of measurement at the other)

2) The necessity of superluminal influences (dependence of particular outcomes at one wing on choice of measurement at the other)

The quantum correlations

$$p(\text{success}) = \frac{1}{4}[\, p(\text{agree}|SS) + p(\text{agree}|ST) + p(\text{agree}|TS) + p(\text{disagree}|TT)]$$

Realist theories that are locally causal predict

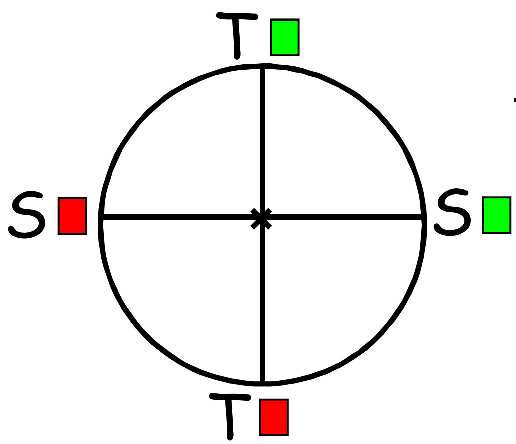
$p(\text{success})$ □ 0.75

A Bell Inequality

Quantum theory predicts that one can achieve

$$p(\text{success}) \simeq 0.85$$

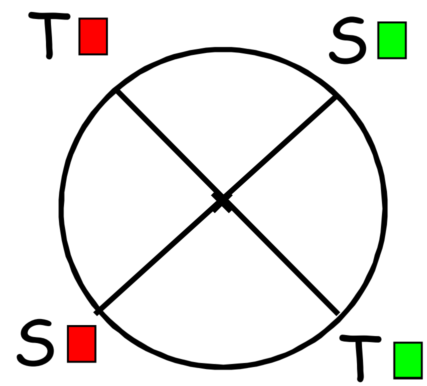
The Bell-inequality violation in quantum theory

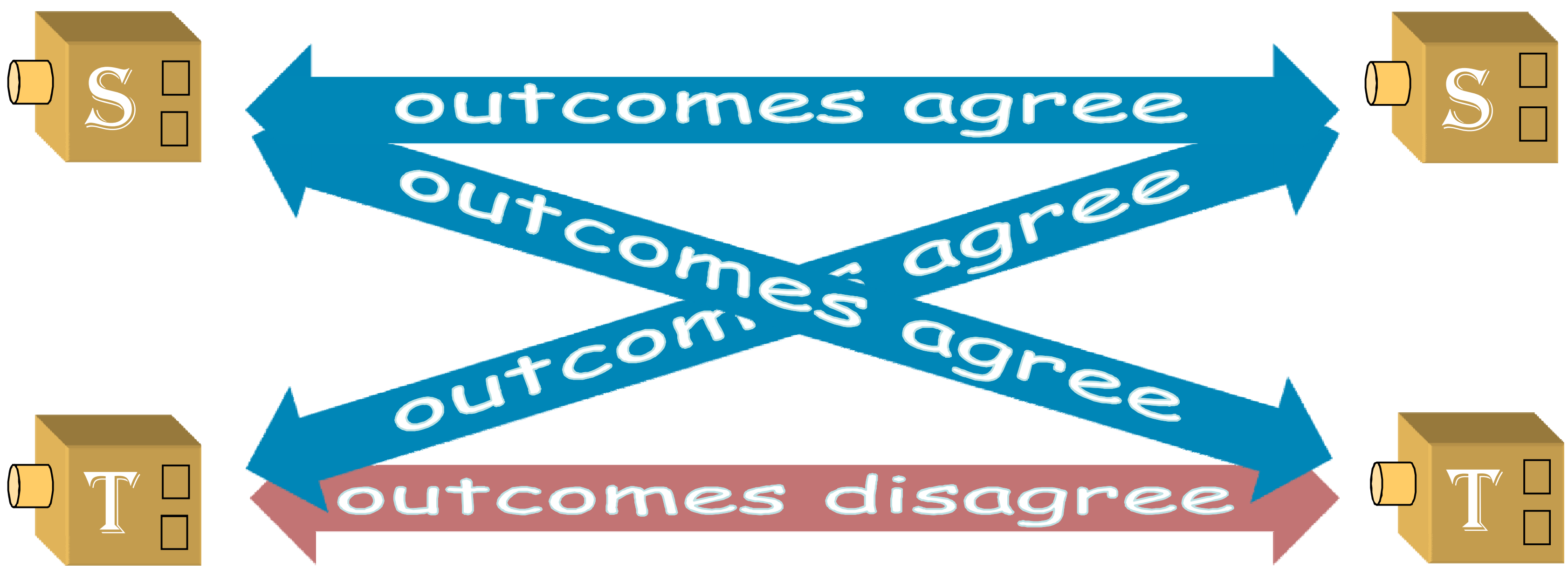


$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

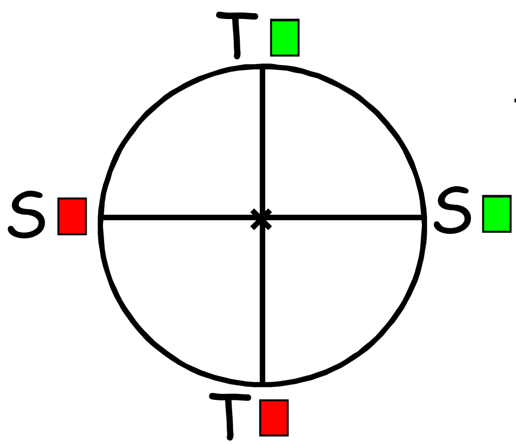
$$\text{p(success)} = \frac{1}{2} + \frac{1}{2\sqrt{2}}$$

$$\simeq 0.85$$





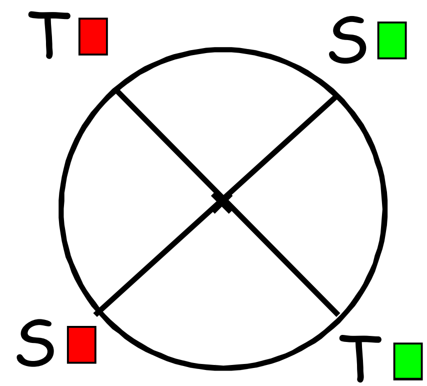
The Bell-inequality violation in quantum theory



$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

$$\mathrm{p(success)} = \frac{1}{2} + \frac{1}{2\sqrt{2}}$$

$$\simeq 0.85$$



$${}_A\langle +\hat{n}|\psi\rangle_{AB} = [\cos(\theta/2)\,{}_A\langle 0| + \sin(\theta/2)\,{}_A\langle 1|]\frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

$$= \cos(\theta/2)|0\rangle_B + \sin(\theta/2)|1\rangle_B$$

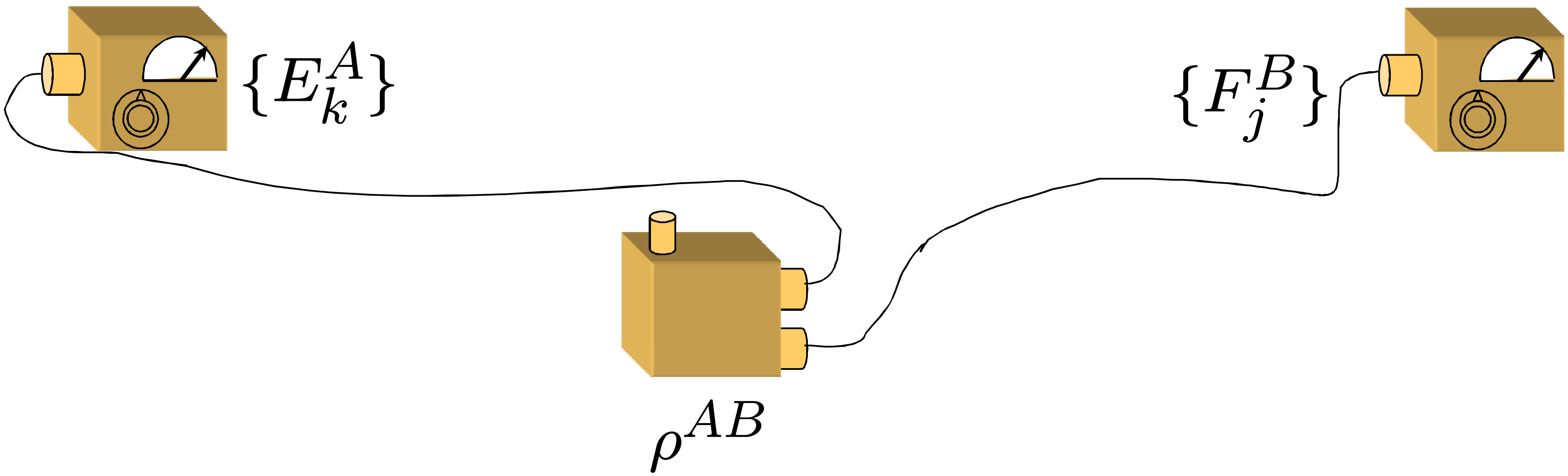
$$= |+\hat{n}\rangle_B$$

$$|\langle +\hat{n}|_A\langle +\hat{m}|_B|\psi\rangle_{AB}|^2 = |\langle +\hat{m}|+\hat{n}\rangle|^2 = \cos^2(\theta/2)$$

$$p(\text{agree}|SS) = p(\text{agree}|ST) = p(\text{agree}|TS) = p(\text{disagree}|TT)$$

$$= \cos^2(\pi/8) = \frac{1}{2} + \frac{1}{2\sqrt{2}}$$

No signalling in quantum theory



$$p(j) = \sum_k p(k,j)$$

$$= \sum_k \mathrm{Tr}_{AB}[\,(E_k^A \otimes F_j^B)\ \rho^{AB}]$$

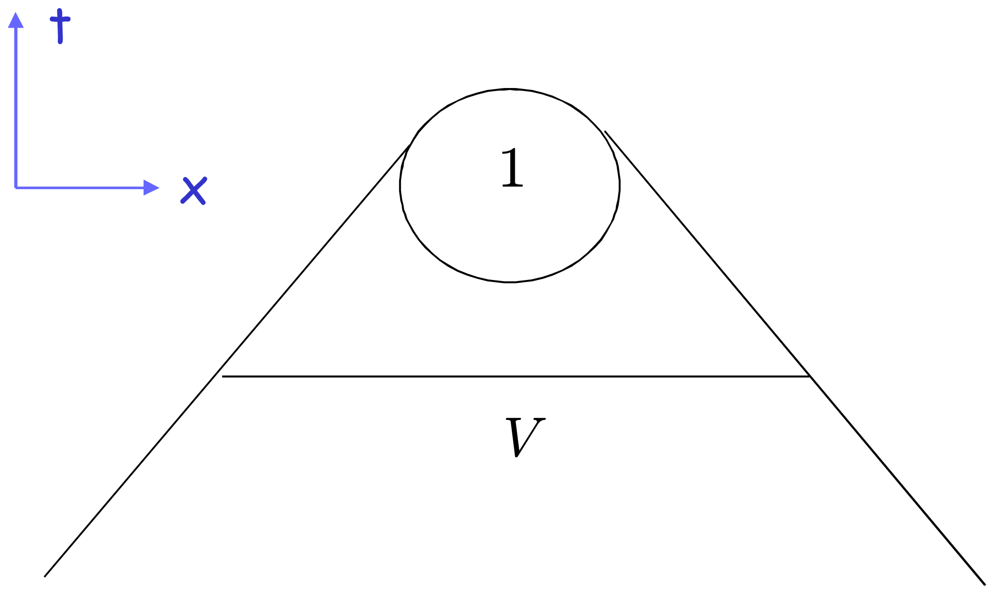
$$= \mathrm{Tr}_{AB}[\,(I^A \otimes F_j^B)\ \rho^{AB}]$$

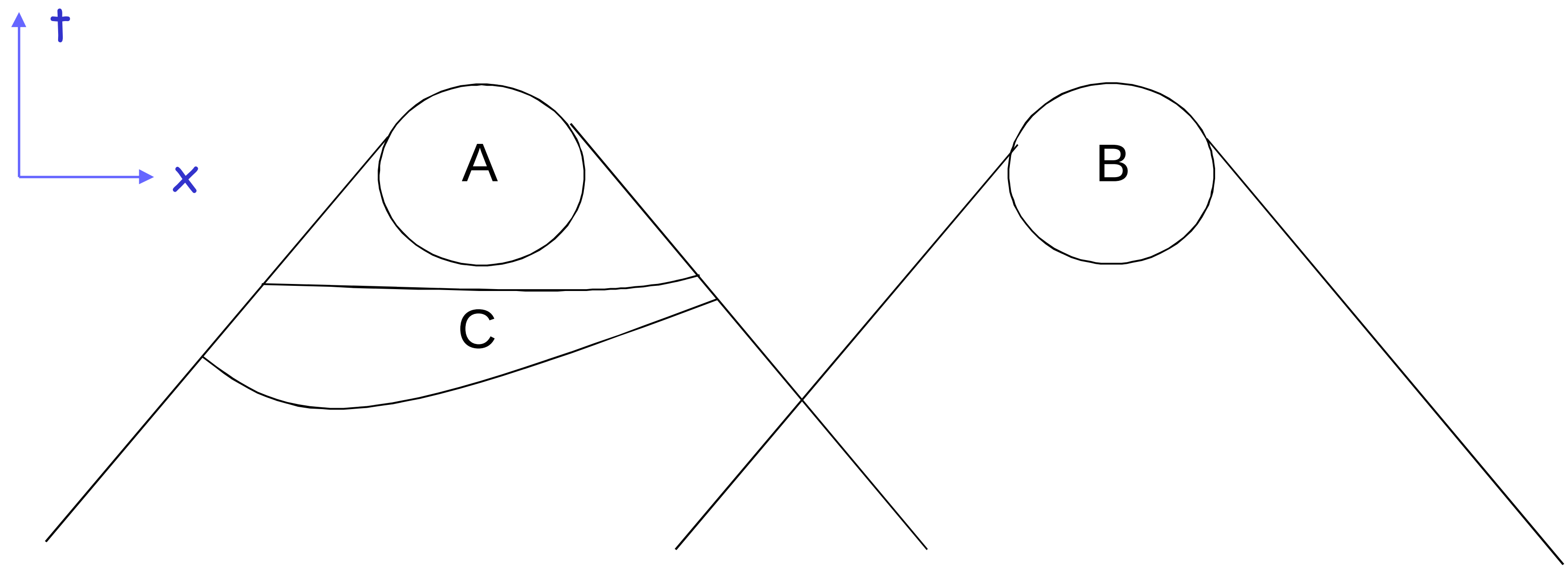
Independent of choice of measurement at A

Note that $[E_k^A, F_j^B] = 0$ for A and B space-like separated

Nonlocality in more depth

"The [beables] in any space-time region 1 are determined by those in any space region V, at some time t, which fully closes the backward light cone of 1. Because the region V is limited, localized, we will say the theory exhibits *local determinism*.
-- J.S. Bell

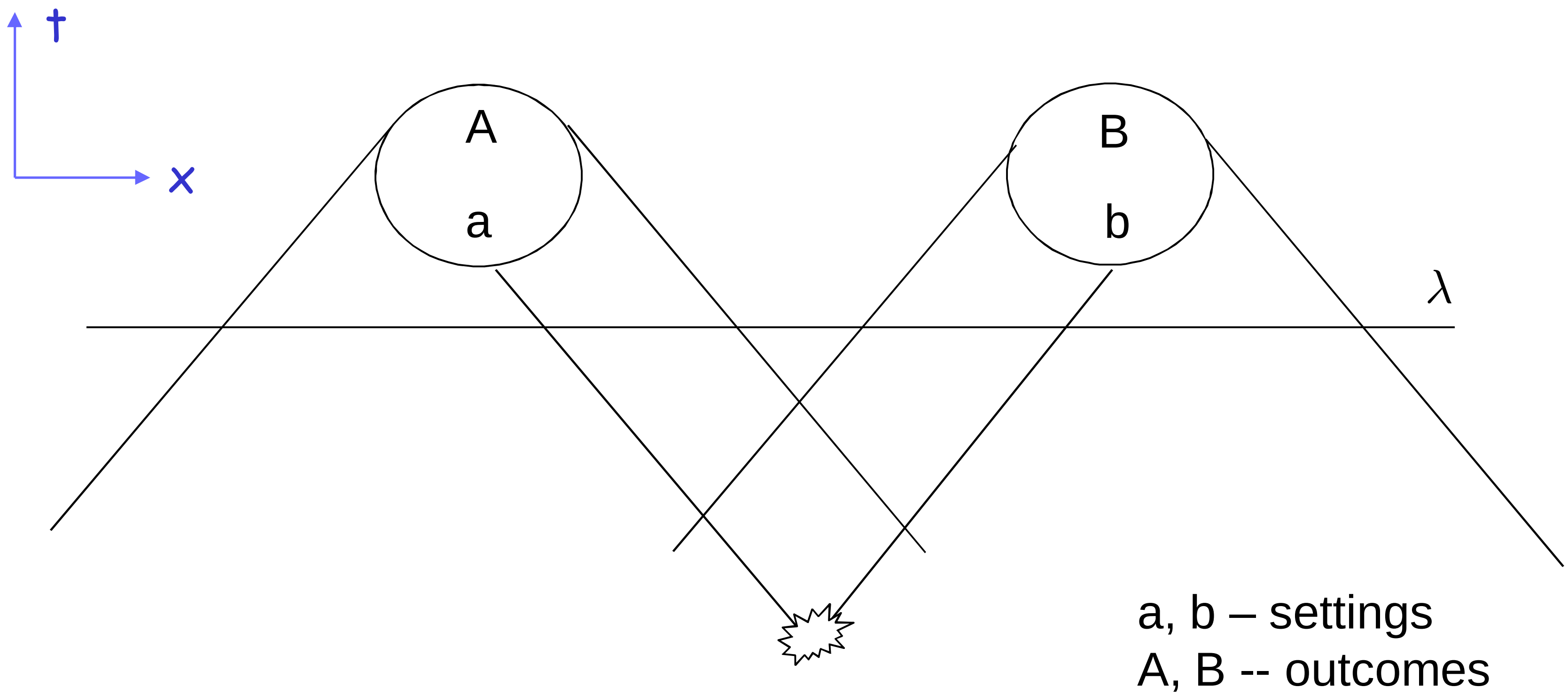




"A theory will be said to be ***locally causal*** if the probabilities for the values of local beables in a space-time region A are unaltered by specification of values of local beables in a space-time region B, when what happens in the backward light cone of A is already sufficiently specified, for example by a full specification of local beables in a space-time region C."
-- J. S. Bell

Local causality

$$p(X_{\mathcal{A}} | X_{\mathcal{B}}, \lambda_{\mathcal{C}}) = p(X_{\mathcal{A}} | \lambda_{\mathcal{C}})$$



Locality causality implies

$$p(A|a,b,B,\lambda) = p(A|a,\lambda)$$
$$p(B|a,b,A,\lambda) = p(B|b,\lambda)$$

and implies *factorizability*

$$p(A,B|a,b,\lambda) = p(A|a,\lambda)p(B|b,\lambda)$$

Factorizability from local causality

Recall Bayes' rule

$$p(A,B) = p(A|B)p(B)$$
$$p(A,B|C) = p(A|B,C)p(B|C)$$

therefore

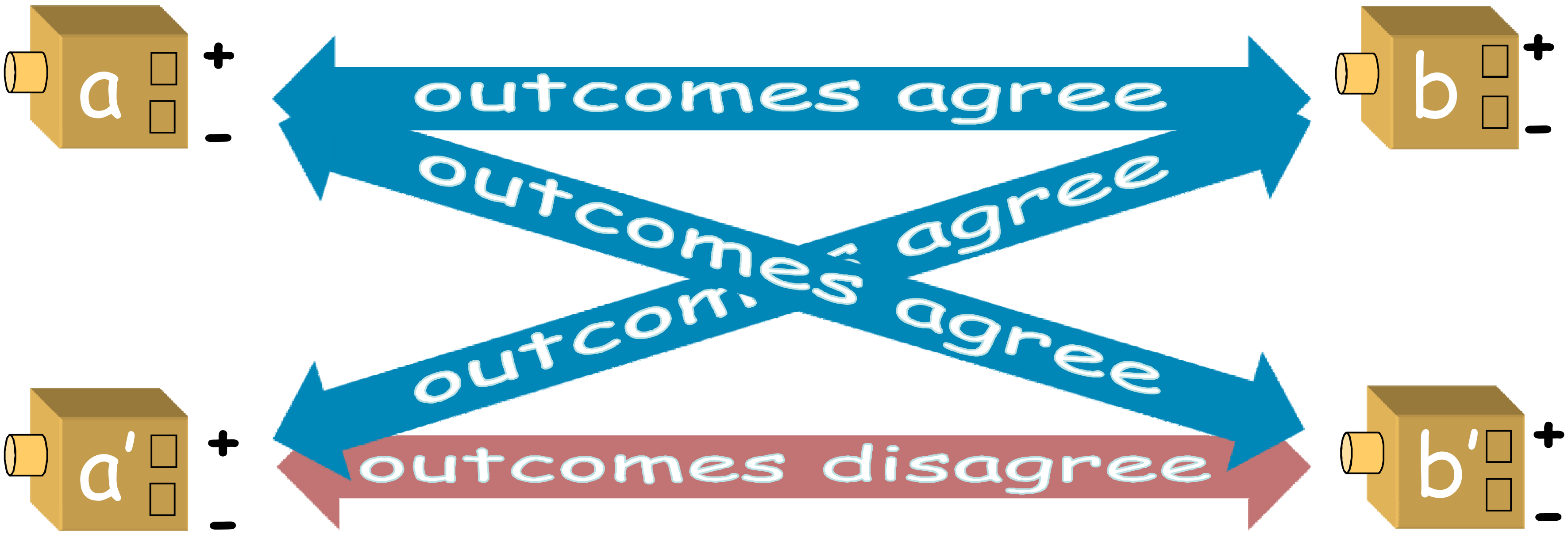
$$p(A,B|a,b,\lambda) = p(A|B,a,b,\lambda)p(B|a,b,\lambda)$$

By local causality

$$p(A|B,a,b,\lambda) = p(A|a,\lambda)$$
$$p(B|a,b,\lambda) = p(B|b,\lambda)$$

Thus

$$p(A,B|a,b,\lambda) = p(A|a,\lambda)p(B|b,\lambda)$$



$$\tfrac{1}{4}[\, p(\text{agree}|ab) + p(\text{agree}|ab') + p(\text{agree}|a'b) + p(\text{disagree}|a'b')] \leq 3/4$$

Define $C(a,b) = (+1)\,p(\text{agree}|ab) + (-1)\,p(\text{disagree}|ab)]$

$$|C(a,b) + C(a',b) + C(a,b') - C(a',b')| \leq 2$$

The Clauser-Horn-Shimony-Holt (CHSH) inequality

These (equivalent) inequalities can be derived from local causality
See e.g. J.S. Bell, Speakable and Unspeakable, Chap. 16, App. 2

Applications of nonlocality

Magic is a natural force that can be used to override the usual laws of nature.
-- Harry Potter entry in wikipedia

Bell-inequality violations are natural phenomena that can be used to override the usual (classical-like) laws of nature

Quantum Spellcraft

Based on Bell-inequality violation

Reduction in communication complexity

Buhrman, Cleve, van Dam, SIAM J.Comput. 30 1829 (2001)
Brassard, Found. Phys. 33, 1593 (2003)

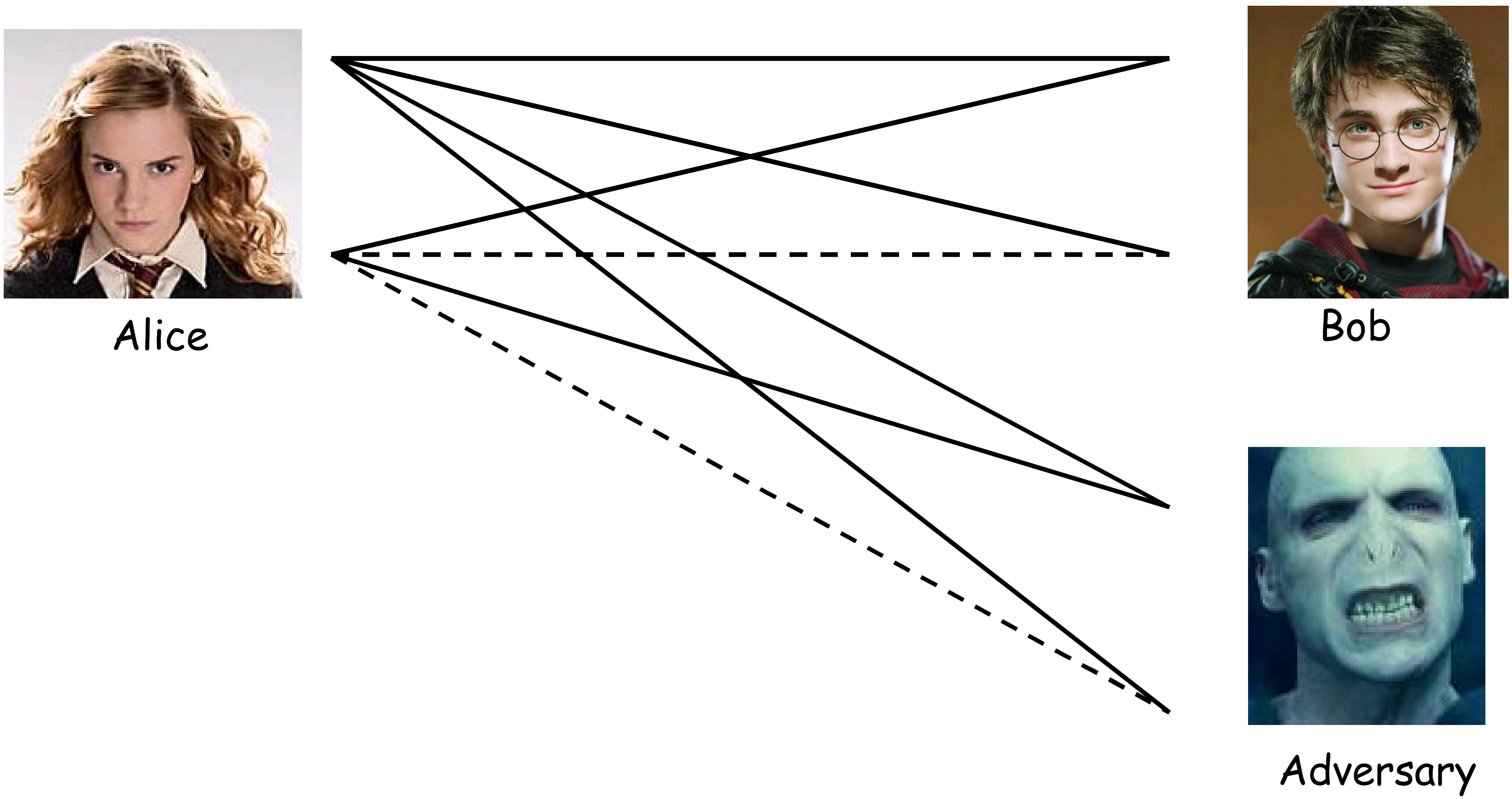
Device-independent secure key distribution

Barrett, Hardy, Kent, PRL 95, 010503 (2005)
Acin, Gisin, Masanes, PRL. 97, 120405 (2006)

Enhancing zero-error channel capacity

Cubitt, Leung, Matthews, Winter, arXiv:0911.5300

Monogamy of Bell-inequality violating correlations



Why isn't the world *more* nonlocal?