Graph Theory in Quantum Information

Lecture 3

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Chris Godsil, University of Waterloo

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Graph Isomorphism

- solved in practice (nauty)
- P < graph isom. < NP-complete
- graph isom. for graphs of bounded valency $\in$ P (E. Luks)

X_1 X_2

1–2–3 $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

n

1–3–2 $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

A $n \times n$

X, Y

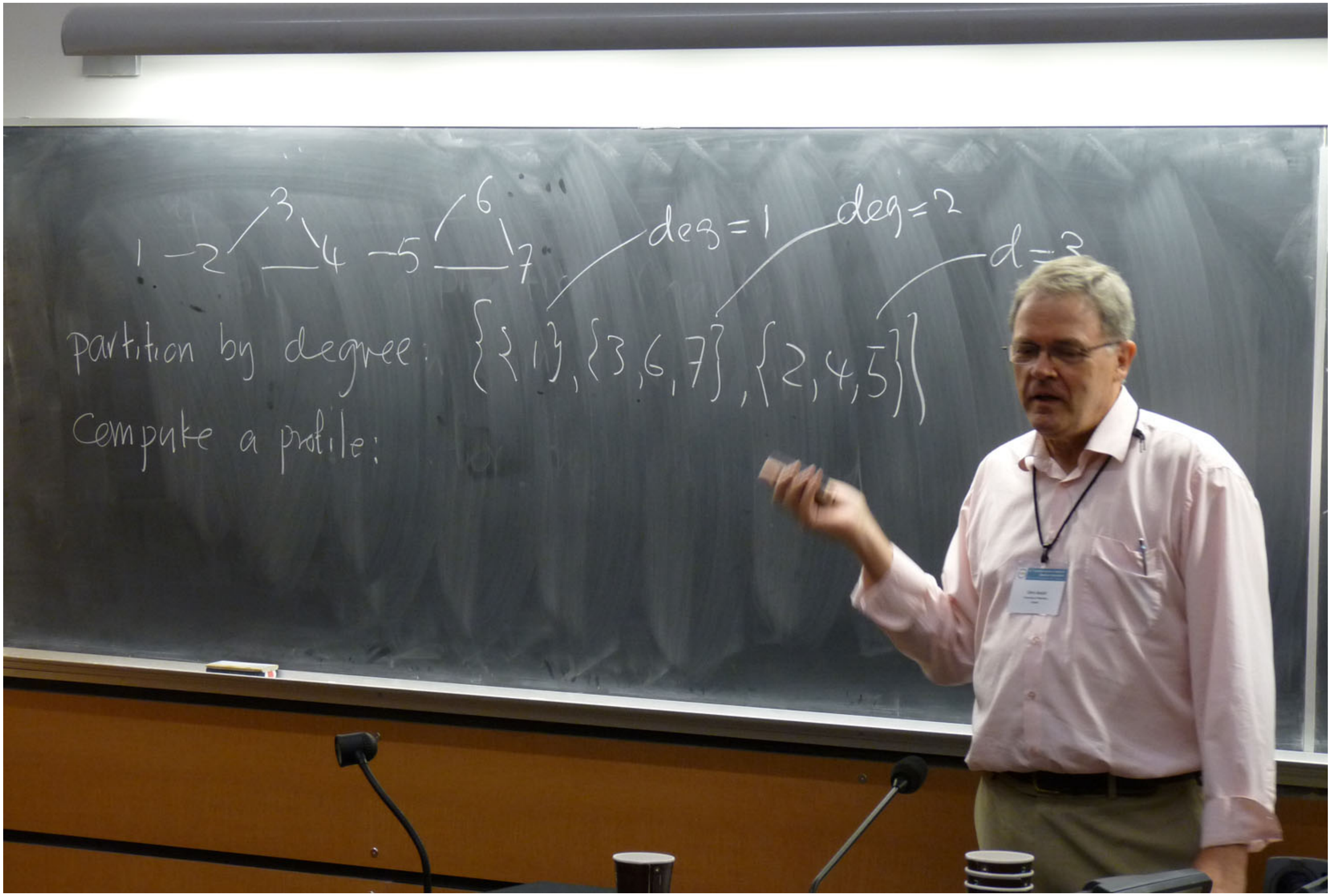
$V(X)$ $V(Y)$

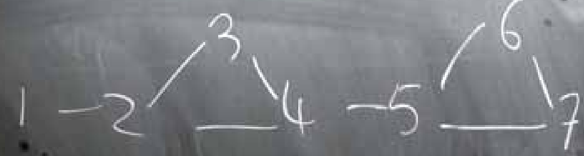
$$P^T A(X) P = A(Y)$$

perm. matrix

Quantum methods

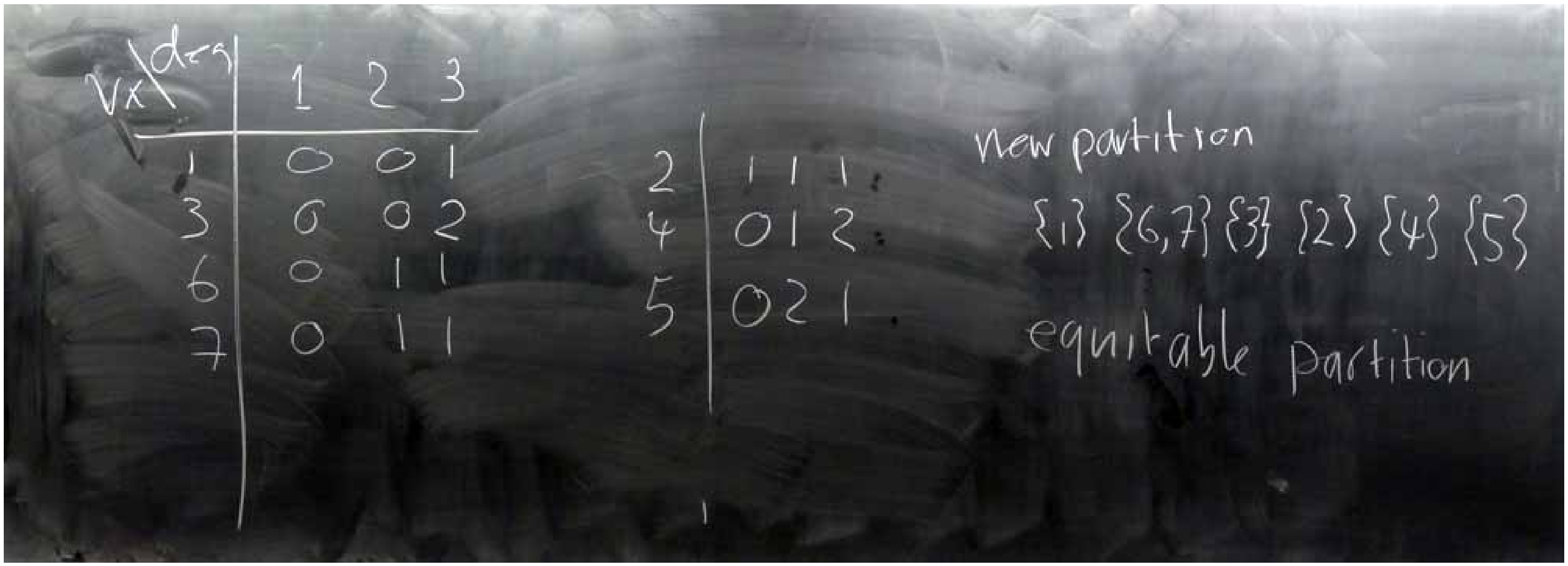
- hidden subgroup problem
- "walks" — continuous or discrete quantum walks
  - bosonic Hubbard model. → symmetric powers


1 — 2 3 4 — 5 6 7
deg = 1
deg = 2
d = 3
partition by degree: {{1}, {3, 6, 7}, {2, 4, 5}}
Compute a profile:



partition by degree: $\{\overbrace{\{1\}}^{\deg=1}, \overbrace{\{3,6,7\}}^{\deg=2}, \overbrace{\{2,4,5\}}^{d=3}\}$

Compute a profile:



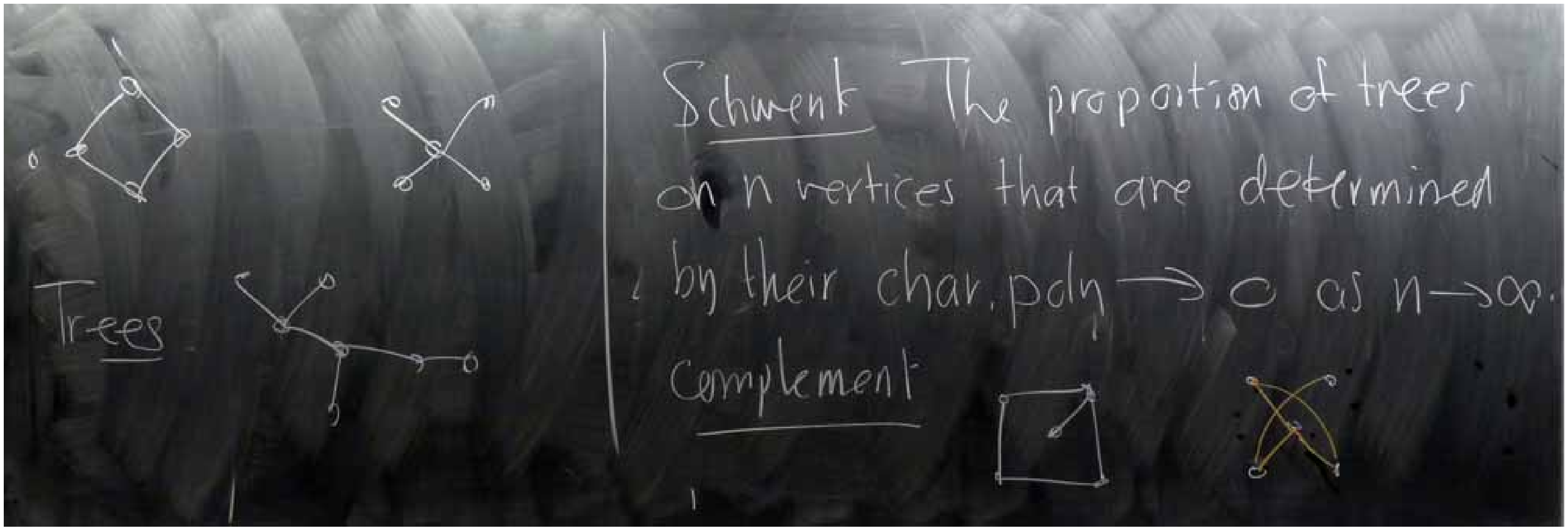
Vx deg
1 2 3
1 0 0 1
3 0 0 2
6 0 1 1
7 0 1 1
2 1 1 1
4 0 1 2
5 0 2 1
new partition
{1} {6,7} {3} {2} {4} {5}
equitable partition


1
regular, degree k
{1} {V(X)\1}

Spectrum

X graph with adjacency matrix A

$\phi(X,t) = \det(tI - A)$ characteristic polynomial


Trees
Schwenk The proportion of trees on n vertices that are determined by their char. poly → 0 as n → ∞.
Complement

Continuous Quantum Walk on a Graph

$$H_X(t) = \exp(iAt) = \sum_{n \geq 0} \frac{i^n t^n}{n!} A^n$$

$$W(X,t) = (I - tA)^{-1} = \sum_{n \geq 0} t^n A^n$$ — walk generating fn

A walk in a graph is a sequence of ve

$v_0, \dots, v_m$

Such that $v_{i-1} \sim v_i$ for $i = 1, \dots, m-1$.

<u>Claim</u> $(A^r)_{i,j}$ = # walks from i to j in X with length r.

$$t^{-1}\,\frac{\phi(X\setminus i, t^{-1})}{\phi(X, t^{-1})} = \big(W(X,t)\big)_{i,i} = \sum_{n\geq 0} t^n \,(A^n)_{i,i}$$

$\phi(X\setminus i, t)$, $\phi(X,t)$ determine the ith diagonal entry of $W(X,t)$ & $H_X(t)$.

$\phi(X,t)$, $\phi(X\setminus i,t)$, $\phi(X\setminus j,t)$ $\phi(X\setminus ij,t)$

determine the ij-entry of $W(X,t)$ & $H_X(t)$!

Latin square graphs

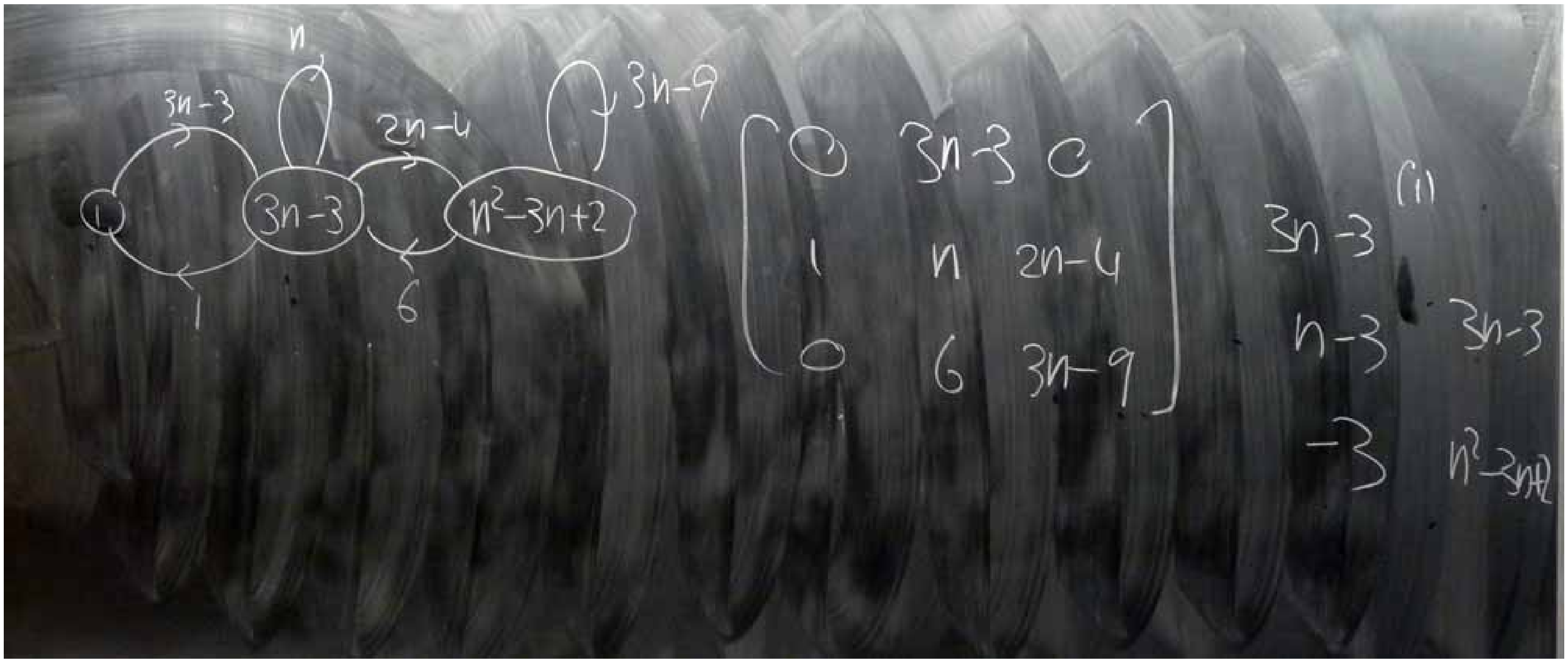
mult. table of a finite group.

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$

$\{(i, j, L_{ij}) : 1 \leq i,j \leq n)$

WS –

LS(8)	2.8×10^5
(9)	1.9×10^{10}
(10)	3.5×10^{16}



n
3n-3
2n-4
3n-9
1
3n-3
n^2-3n+2
1
6
0 3n-3 0
1 n 2n-4
0 6 3n-9
(1)
3n-3
n-3 3n-3
-3 n^2-3n+2