Graph Theory in Quantum Information

Lecture 4

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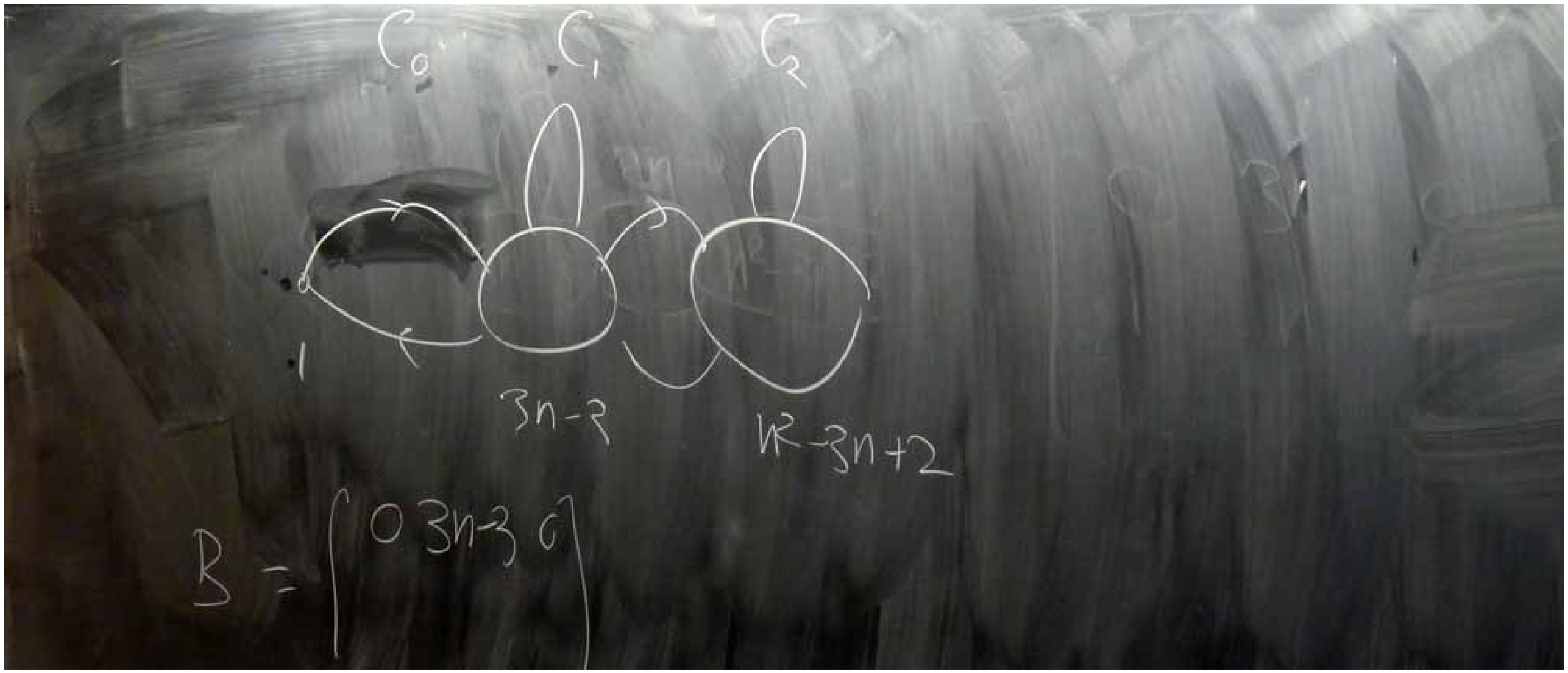

C_0
C_1
C_2
1
$3n-3$
n^2-3n+2
$B = [0 \; 3n-3 \; 0]$

<u>Claim</u> Let X be a Latin square graph and let Y be an induced subgraph of X. Then the char. poly of $X \setminus Y$ is determined the char. poly of Y & its complement.

Corollary $\phi(X \setminus i,j,t)$ only depends on whether i & j are adjacent.

Symmetric powers

input X $\quad X^{\{k\}}$ vxs – subsets of size k from $V(X)$

edges – $\alpha \sim \beta \Leftrightarrow$ symmetric difference $\alpha \oplus \beta$ of α & β is an edge of X.

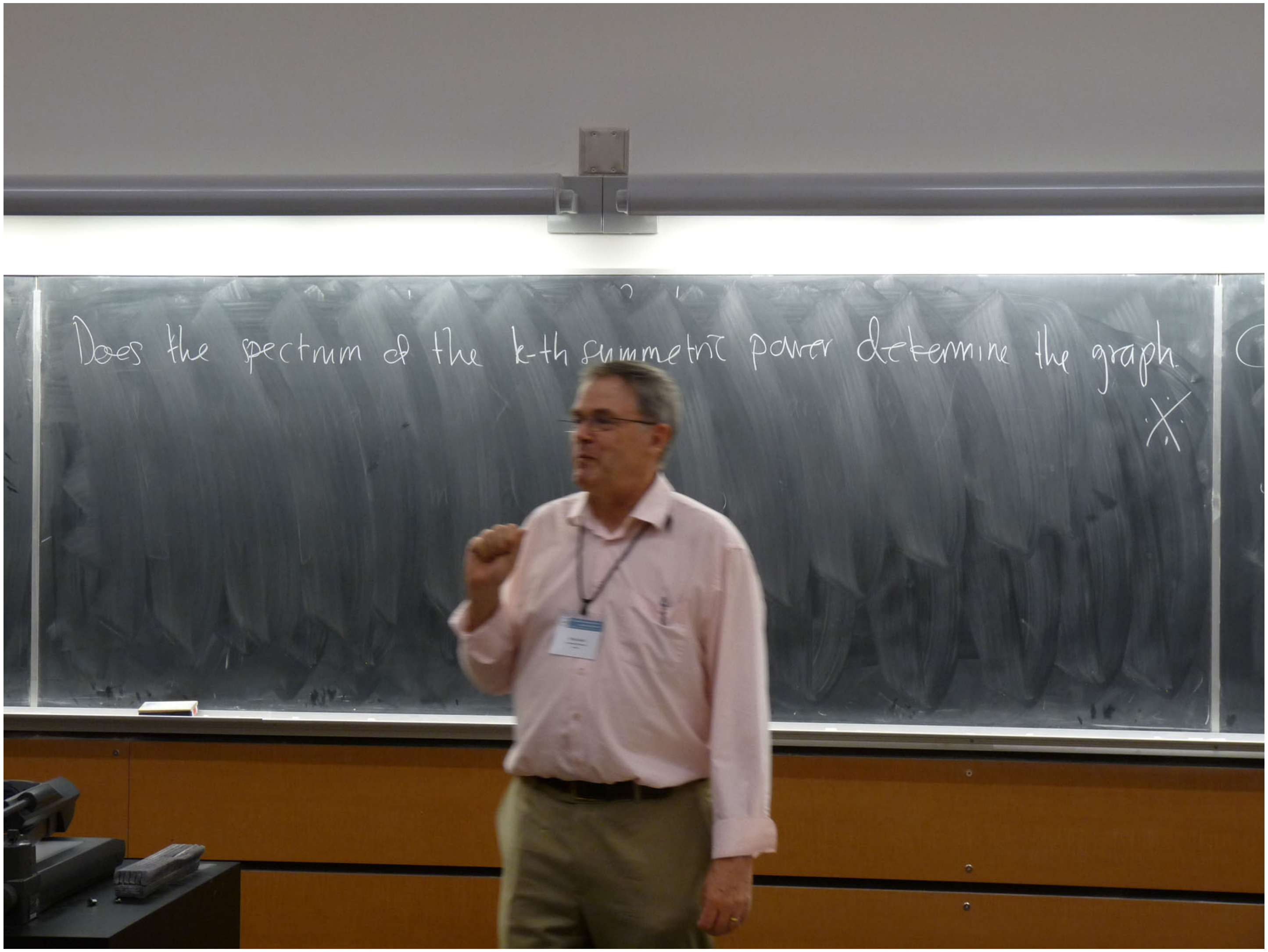
Symmetric powers

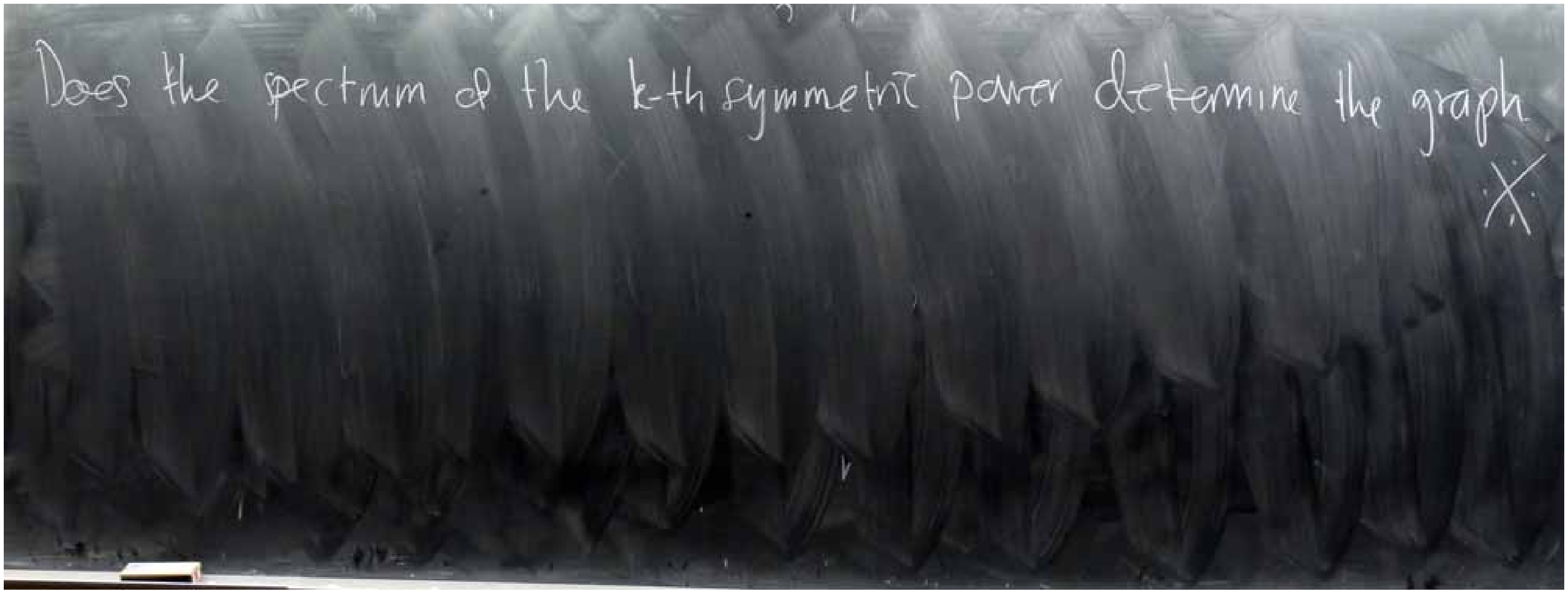
Cai Fuhrer Immerman

input X $\quad$ $X^{\{k\}}$ $-\log_2(|V|)$

vxs – subsets of size k from $V(X)$

edges – $\alpha \sim \beta \iff$ symmetric difference $\alpha \oplus \beta$ of α & β is an edge of X.


Does the spectrum of the k-th symmetric power determine the graph.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Discrete walks An arc in a graph is an ordered pair of adjacent vxs

(1,2) (2,1)

Line digraph of X

vxs – arcs of X

"edges" $((i,j),(j,k))$ $i \sim j$ in X

$j \sim k$

e.g. $LS(4)$ 16 vxs degree 9

Emms' et al

Suppose X is regular with degree k. Let $U = \frac{2}{k}A(LD) - R$

orthogonal

$S^+(M)$

$S^+(U)$ $\quad S^+(U^2)$ $\quad S^+(U^3)$

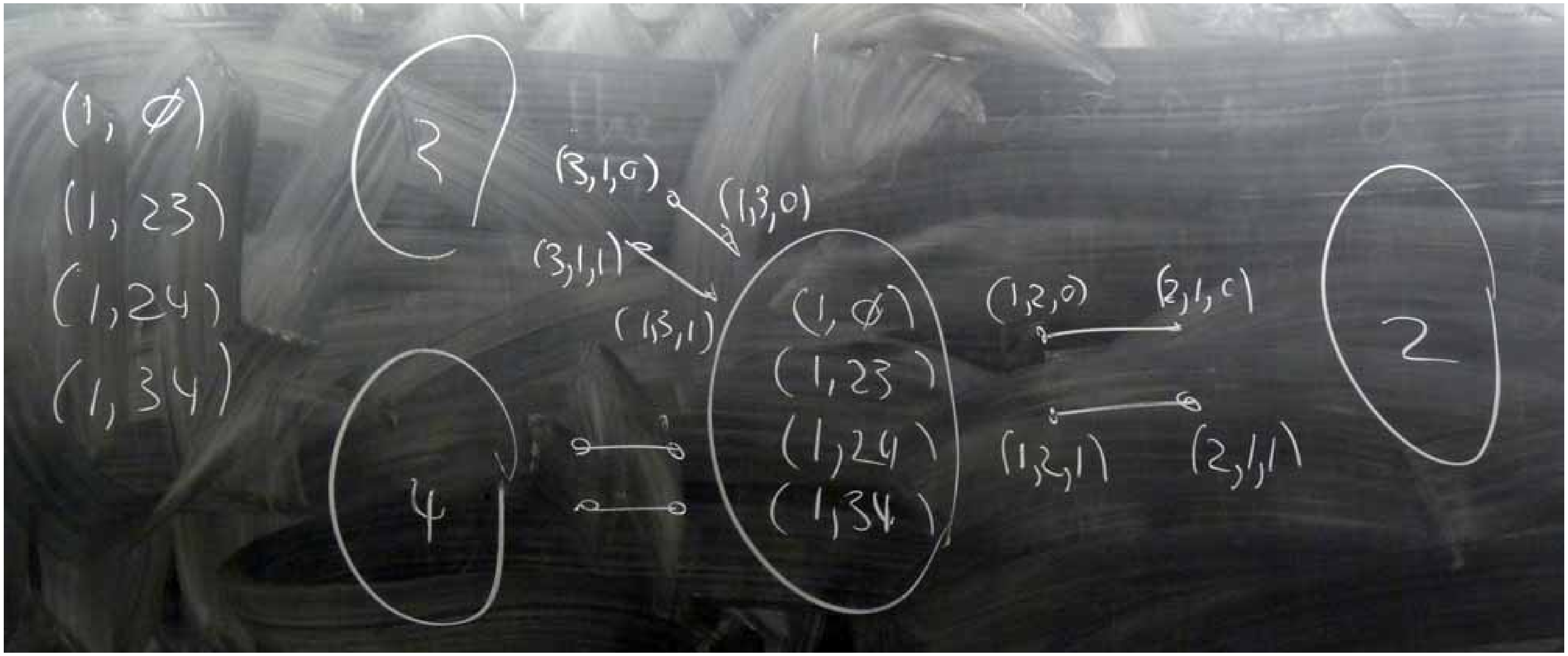
all strongly regular graphs on up to 40 vxs are determined by the spectrum of $S^+(U^3)$

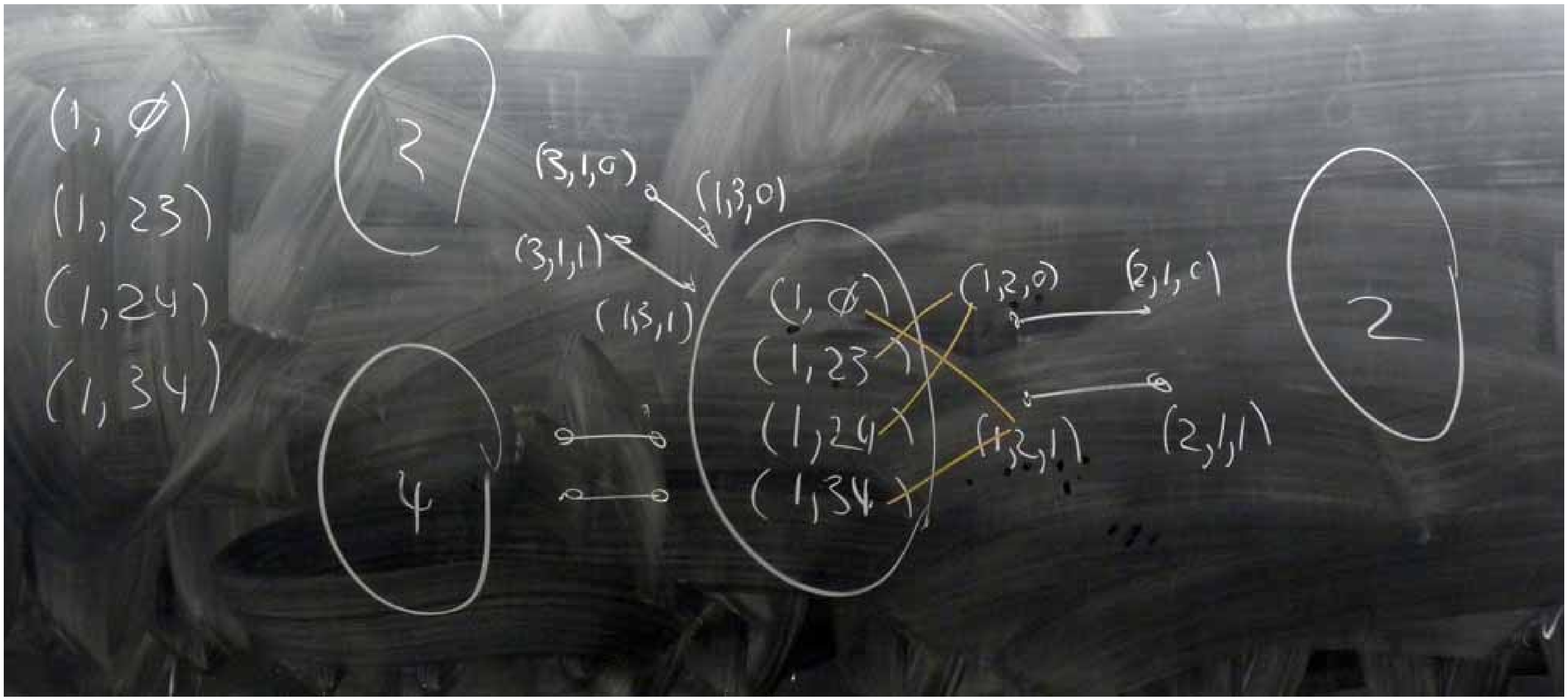
Cai-Fuhrer-Immerman

input X - min valency ≥ 3

$VA = \{ (v,\alpha) : v \in V(X)$
α is a subset of the nbrs of v with even size $\}$

$VB = \{ (v,w,i)$ $v \sim w$ in $V(X)$
$i = 0,1$ $\}$


(1, ∅)
(1, 23)
(1, 24)
(1, 34)
3
(3,1,0)
(1,3,0)
(3,1,1)
(1,3,1)
4
(1, ∅)
(1, 23)
(1, 24)
(1, 34)
(1,2,0)
(2,1,0)
(1,2,1)
(2,1,1)
2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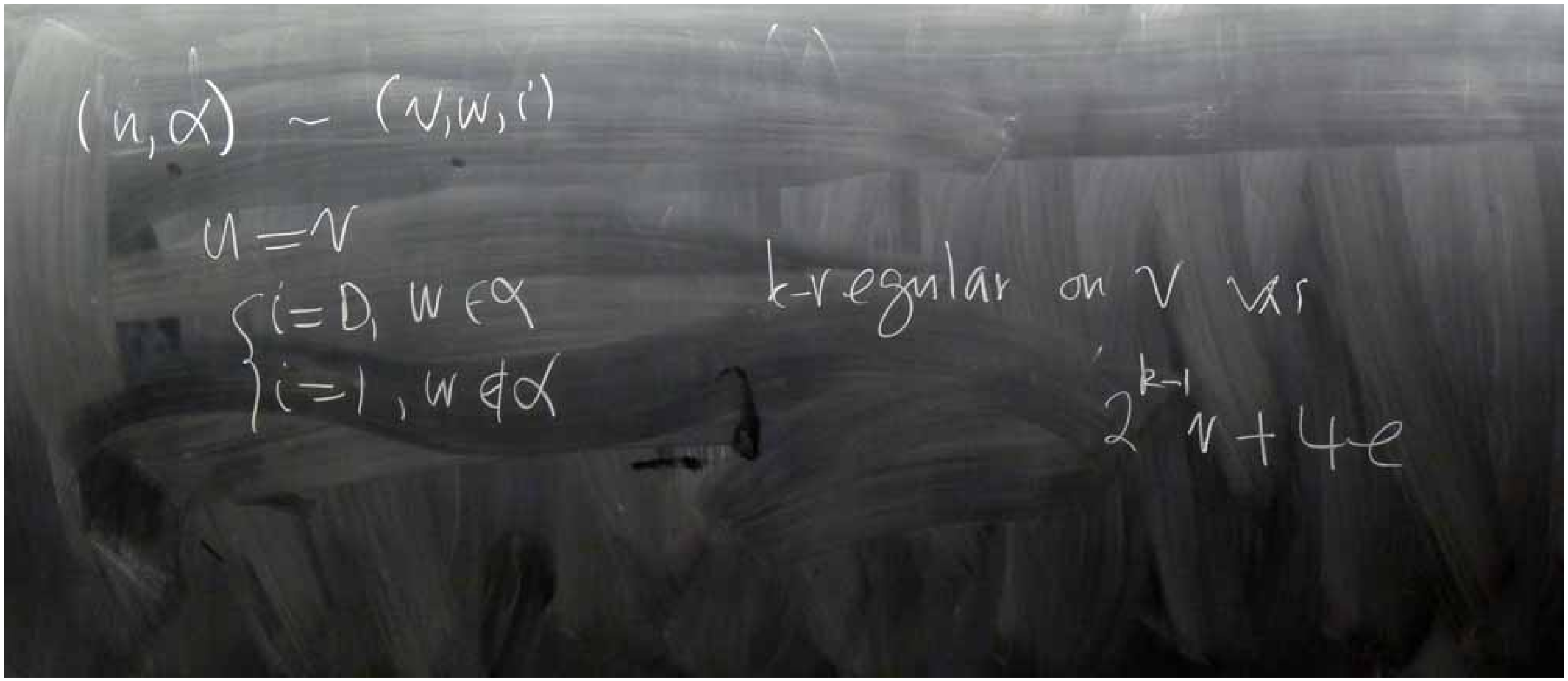
$(u, \alpha) \sim (v, w, i)$
$u = v$
$\begin{cases} i = 0, & w \in \alpha \\ i = 1, & w \notin \alpha \end{cases}$
k-regular on V vxs
$2^{k-1} v + 4e$