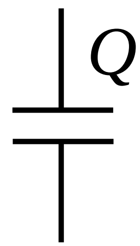
Quantum Annealing With Superconducting Qubits

Mohammad Amin
D-Wave Systems Inc.

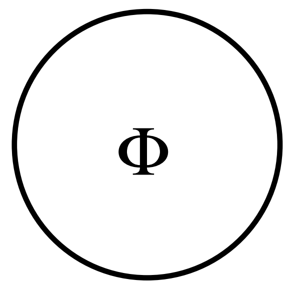
Basic Circuit Elements

Capacitance:



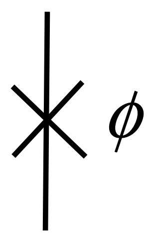
$$E = \frac{1}{2} C V^2 = \frac{Q^2}{2C}$$

Inductance:



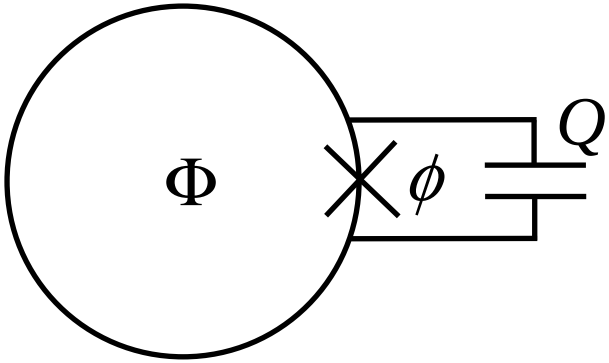
$$E = \frac{1}{2} L I^2 = \frac{\Phi^2}{2L}$$

Josephson junction:



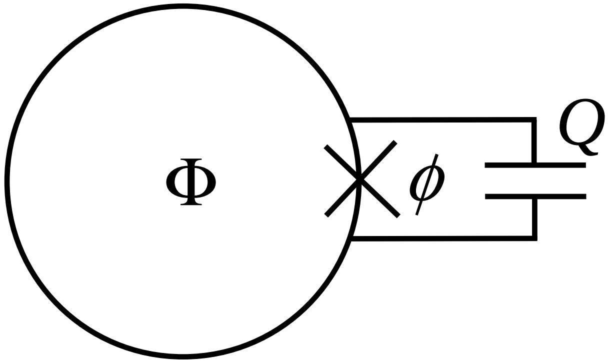
$$E = -E_J \cos\phi$$

RF-SQUID



$$E = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} - E_J \cos\phi$$

RF-SQUID



$$E = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} - E_J \cos\phi$$

Flux quantization: $\phi = \dfrac{2\pi\Phi}{\Phi_0}$ **Faraday's law:** $\dot{\Phi} = V = \dfrac{Q}{C}$

$$\Rightarrow \quad E = \frac{1}{2} C\dot{\Phi}^2 + V(\Phi), \qquad V(\Phi) = \frac{\Phi^2}{2L} - E_J \cos\frac{2\pi\Phi}{\Phi_0}$$

Newtonian mechanics: $E = \boxed{\frac{1}{2} m\dot{x}^2} + \boxed{V(x)}$

Kinetic energy **mass** **Potential energy**

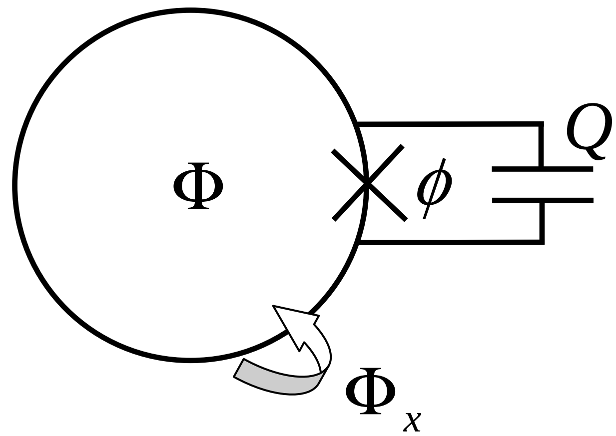
Comparing RF-SQUID with a particle

	particle	SQUID
position	x	Φ
mass	m	C
momentum	$p = m\dot{x}$	$Q = C\dot{\Phi} = CV$
commutators	$[x, p] = i\hbar$	$[\Phi, Q] = i\hbar$
kinetic energy	$\frac{1}{2}m\dot{x}^2 = \frac{p^2}{2m}$	$\frac{1}{2}C\dot{\Phi}^2 = \frac{Q^2}{2C}$
Potential energy	$V(\Phi)$	$V(x)$

RF-SQUID Hamiltonian

$$H = \frac{Q^2}{2C} + V(\Phi)$$

$$V(\Phi) = \frac{(\Phi - \Phi_x)^2}{2L} - E_J \cos\frac{2\pi\Phi}{\Phi_0}$$



RF-SQUID Hamiltonian

$$H = \frac{Q^2}{2C} + V(\Phi)$$

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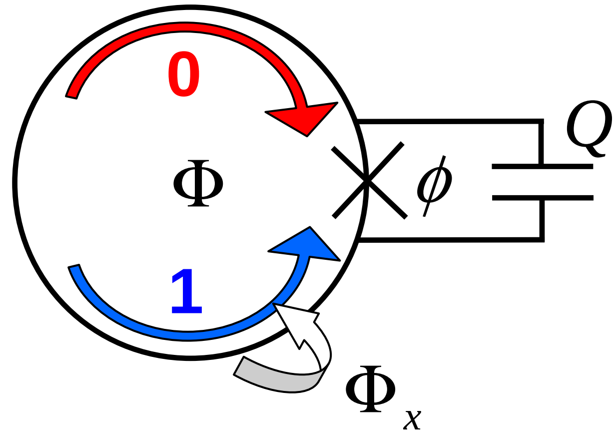
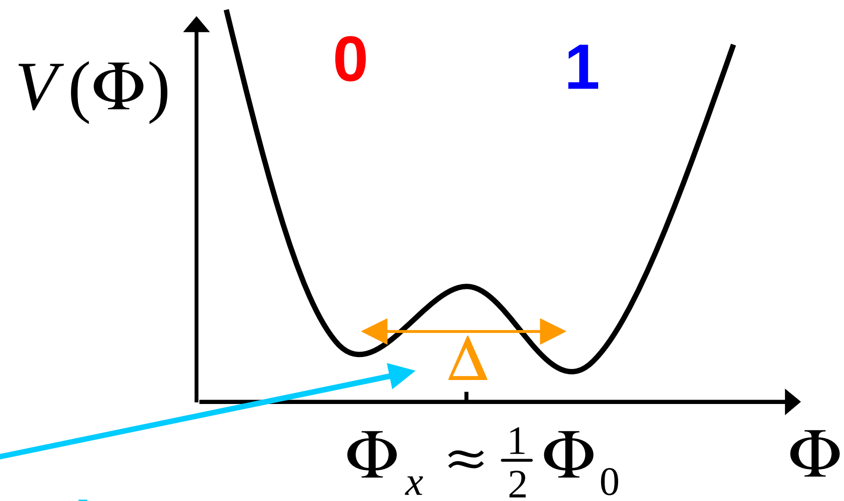
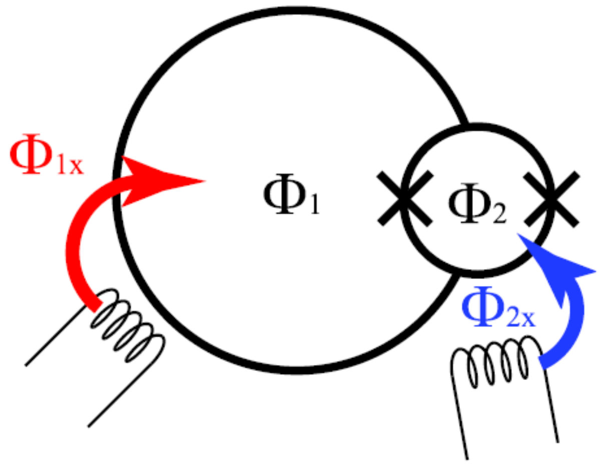


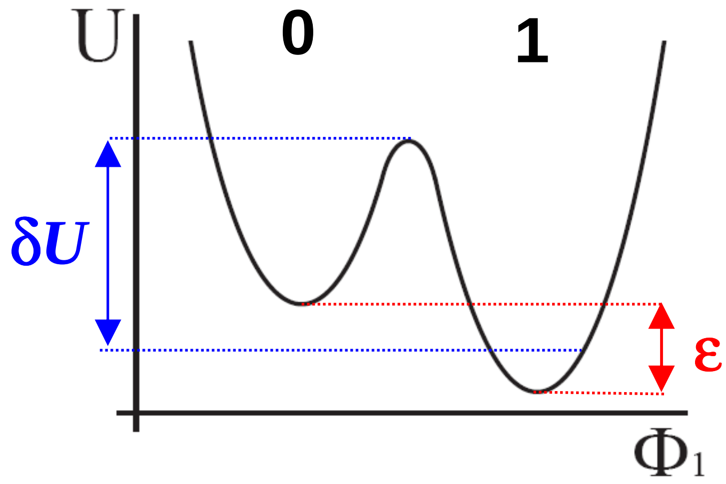
$$H \approx -\frac{1}{2}\left[\varepsilon\sigma_z + \Delta\sigma_x\right]$$



tunneling amplitude

CJJ RF-SQUID Qubit





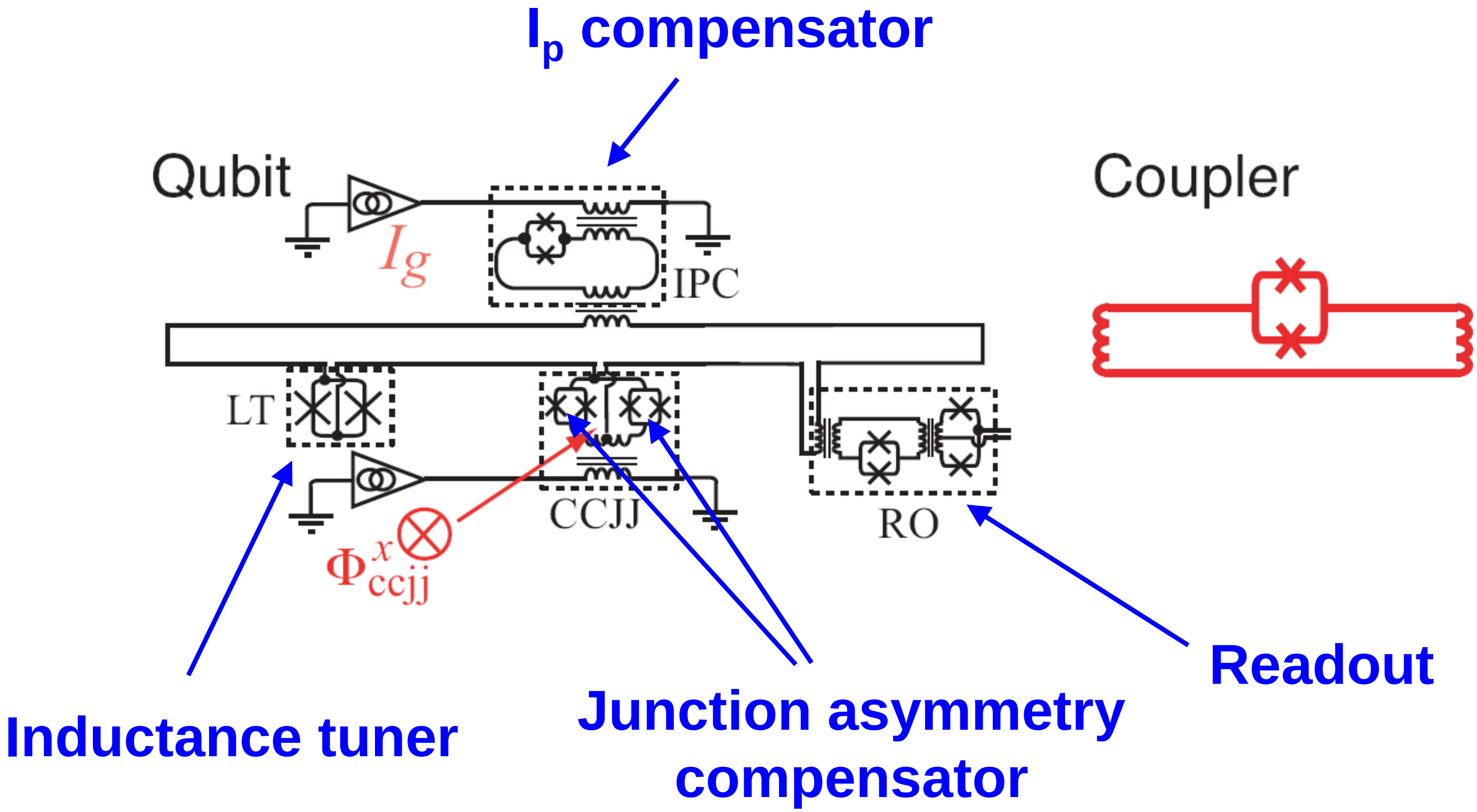
$$H = -\frac{1}{2}\left[\varepsilon\sigma_z + \Delta(\delta U)\sigma_x\right]$$

Control knobs: **Φ_{1x} controls energy bias ε**

Φ_{2x} controls barrier height δU

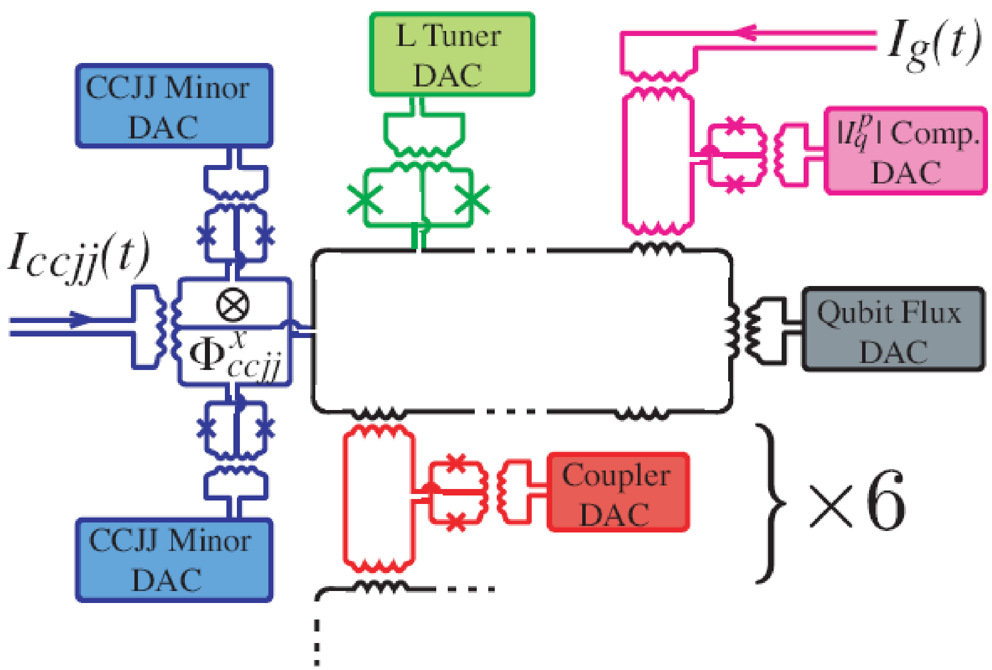
Actual Qubit

Harris *et al*, arXiv:1004.1628 (2010)

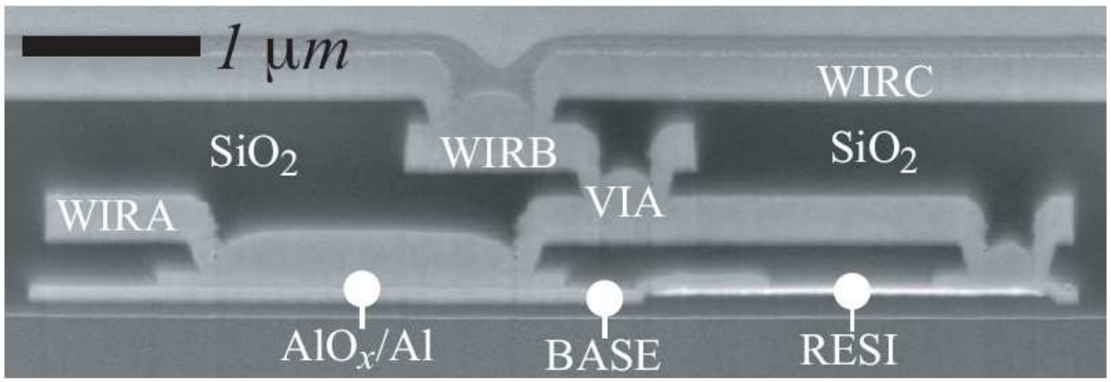


Programmable Magnetic Memory (PMM)

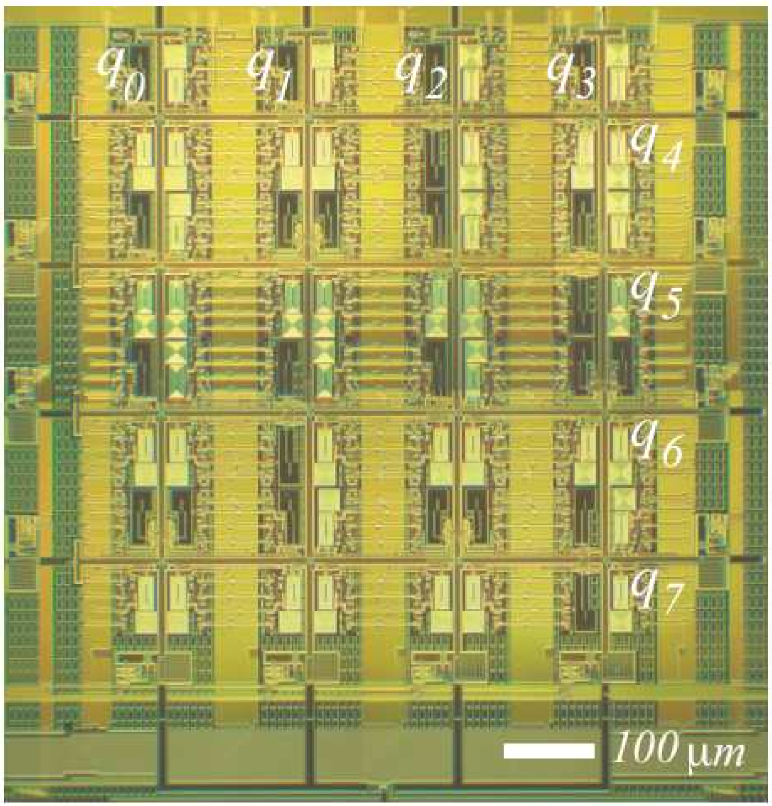
Johnson *et al.*, Supercond. Sci. Technol. 23, 065004 (2010)

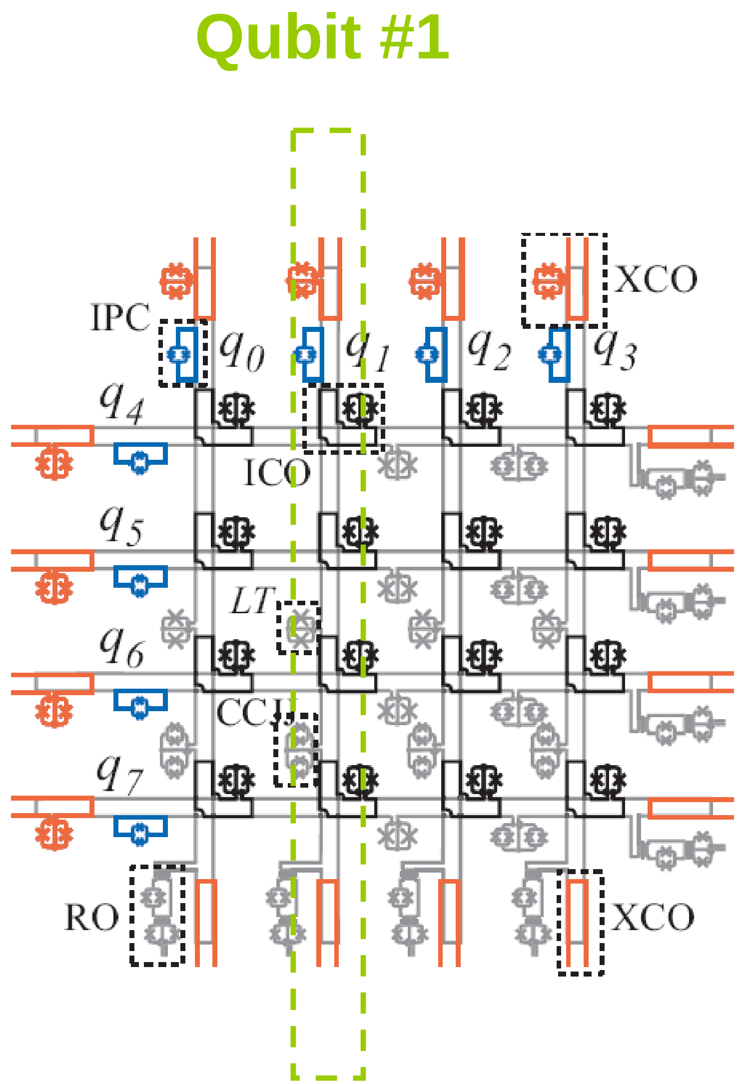


Fabrication

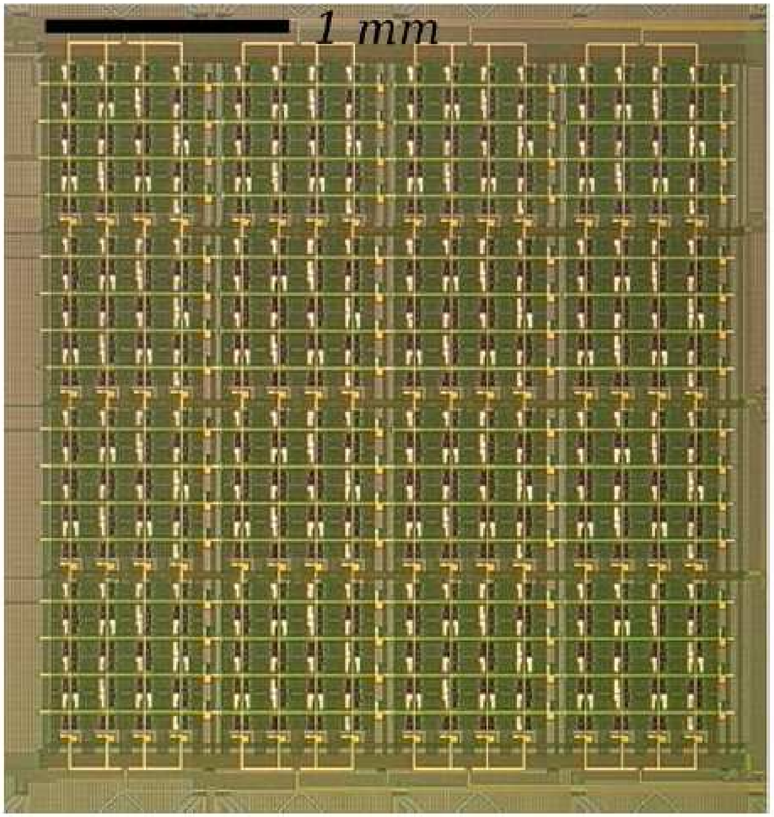


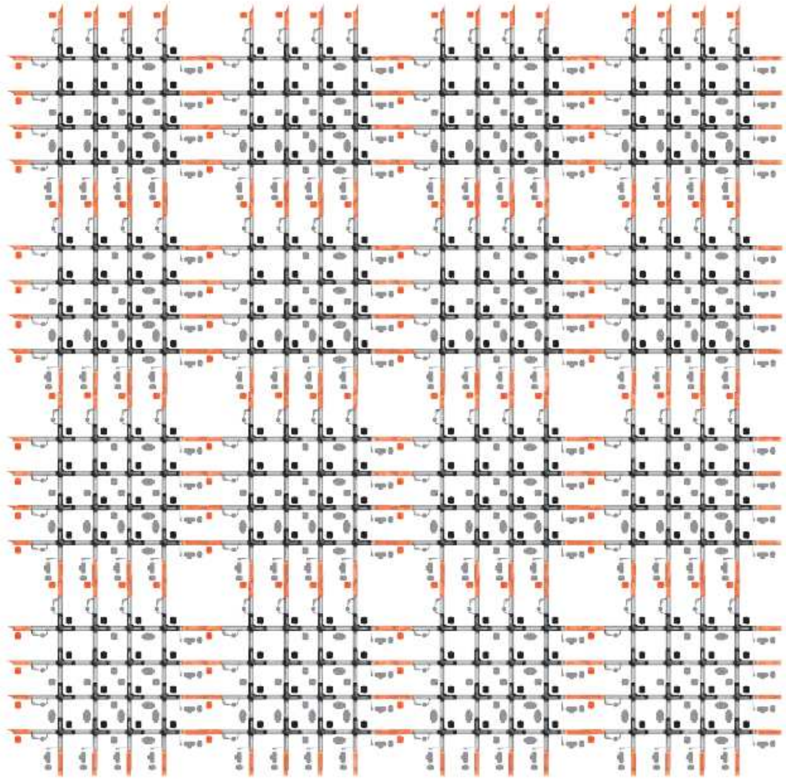
Eight-Qubit Unit Cell



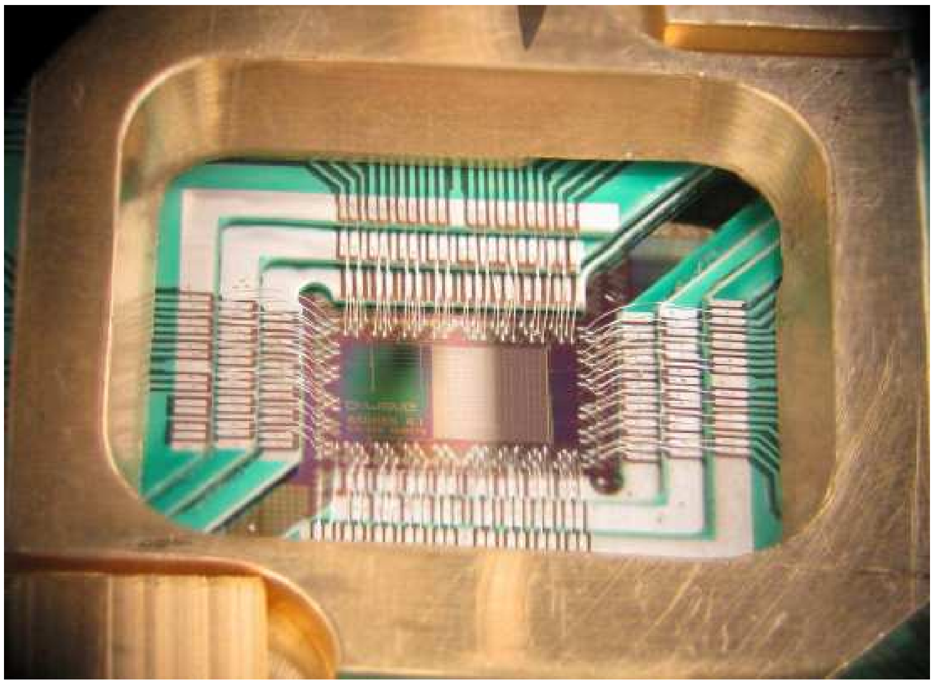


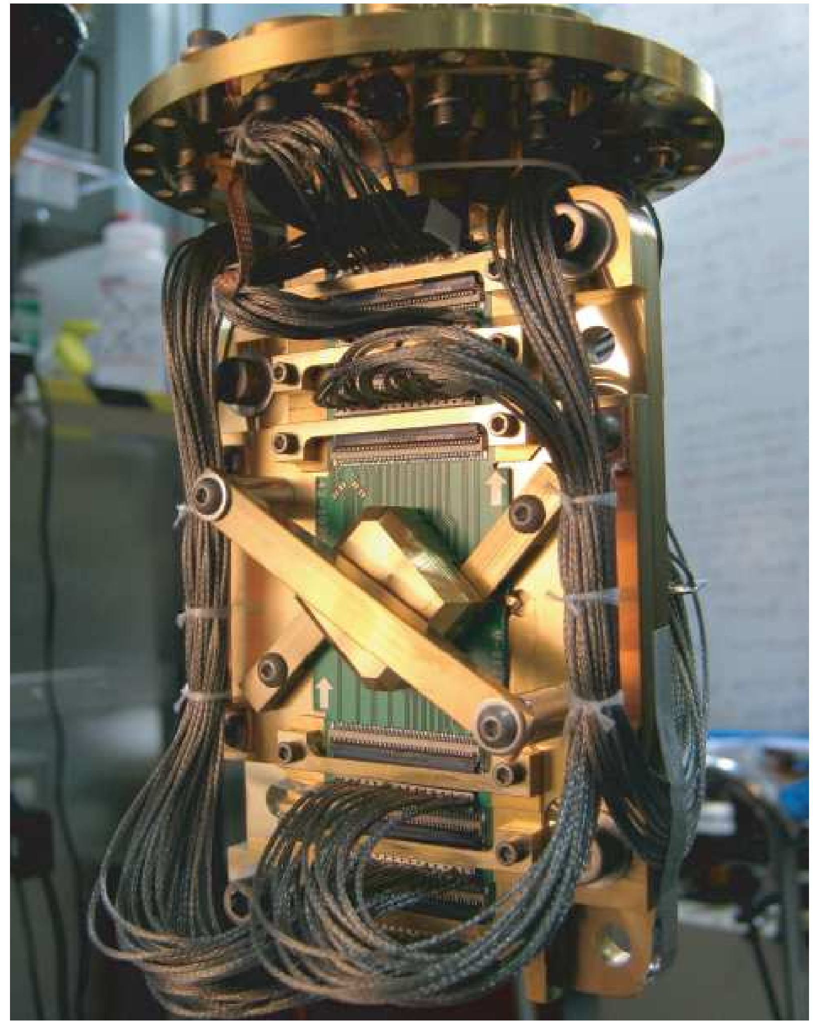
128 Qubits Chip



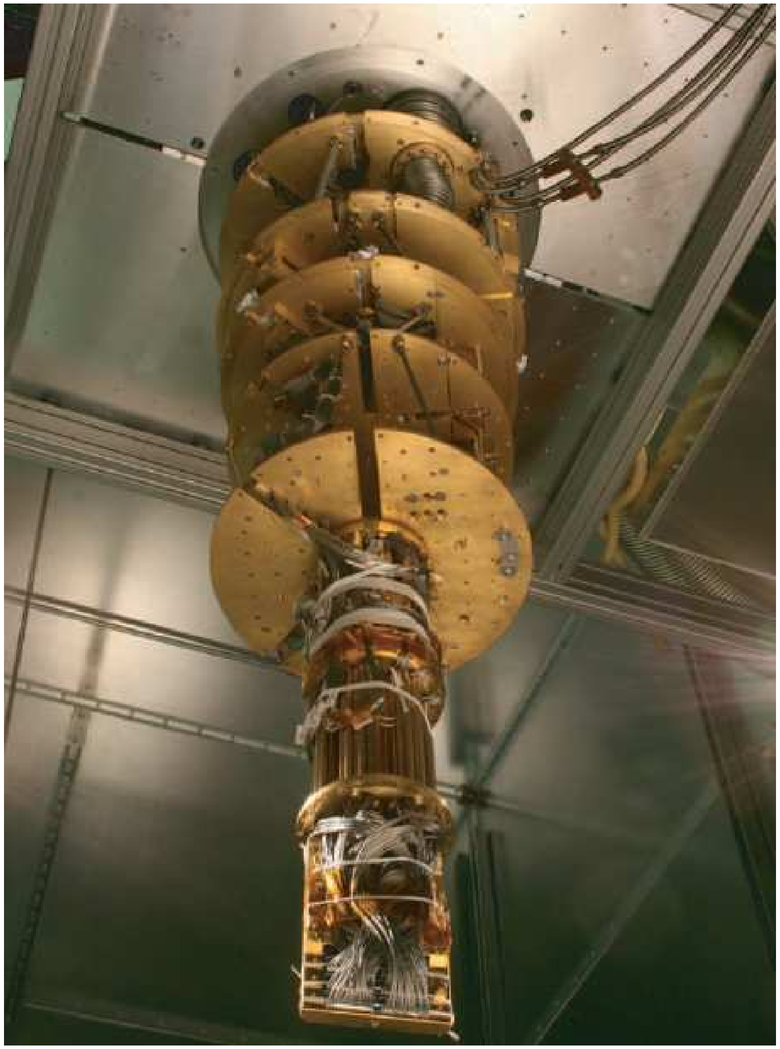


Wire Bounded Chip



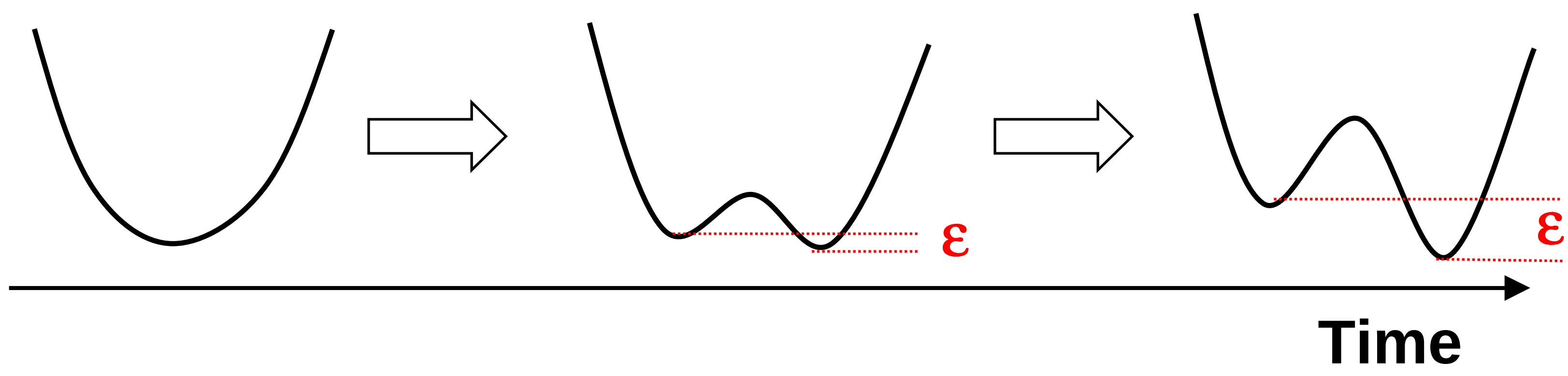


Pulse Tube Dilution Refrigerator



Annealing Process

Annealing happens by raising barrier height ΔU, which provides $\Gamma(t)$

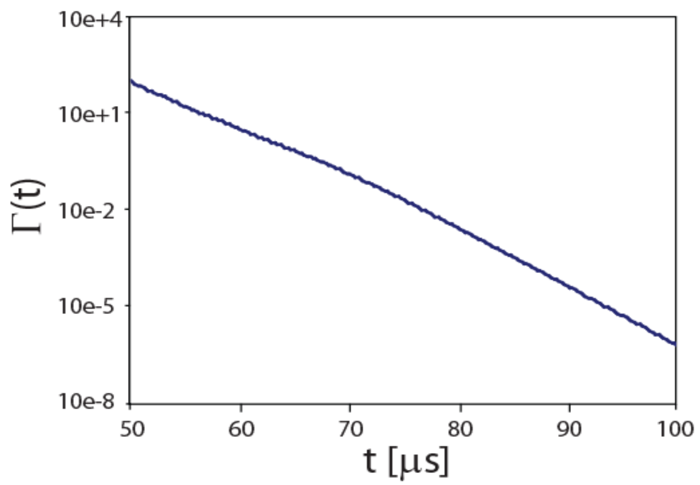


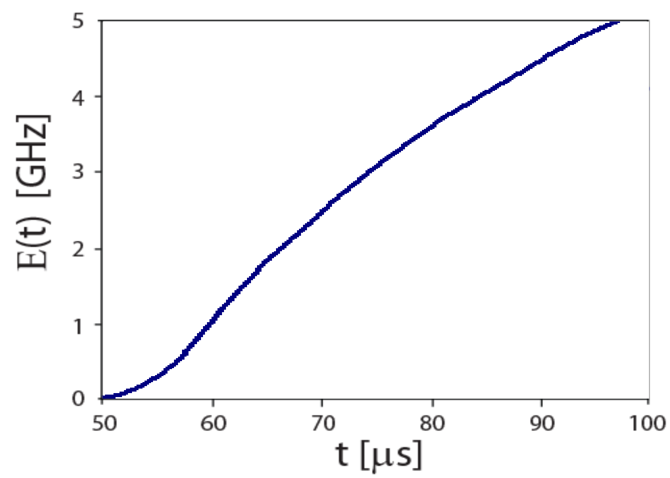
At fixed external flux bias Φ_{1x}, raising barrier height will also changes the energy bias ε

Annealing in Our Hardware

$$H_P = \sum_{i=1}^{N} h_i \sigma_i^z + \sum_{i,j=1}^{N} J_{ij} \sigma_i^z \sigma_j^z$$

Hamiltonian: $$H = E(t)\left(H_P - \Gamma(t) \sum_{i=1}^{N} \sigma_i^x \right) \qquad \Gamma(t) = \frac{\Delta(t)}{2E(t)}$$





$\Gamma(t)$ changed from Γ_{max} to 0: **Quantum annealing!**

$E(t)$ changes from 0 to E_{max}: **Classical annealing!**

Question:

Is it quantum or classical annealing?

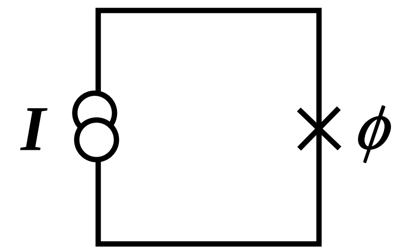
Answer:

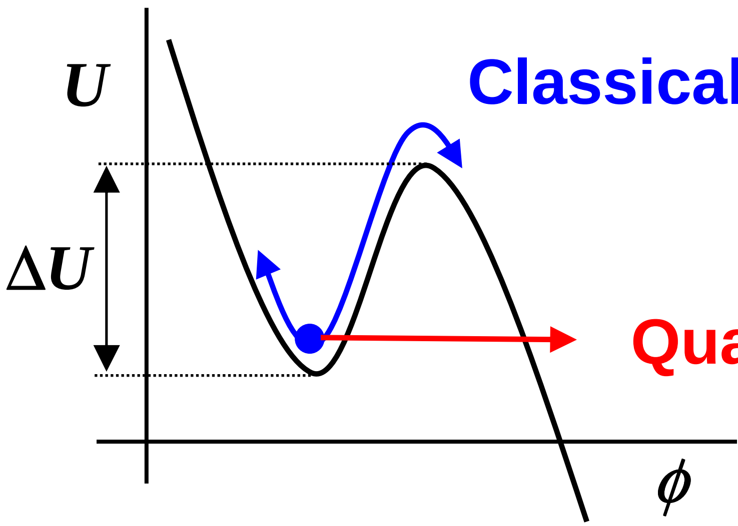
It depends on which one of quantum or thermal fluctuations are stronger

How can we determine this experimentally?

Macroscopic Quantum Tunneling

Current biased Josephson junction:





Classical escape rate:

$$\Gamma = (\omega_p / 2\pi)\exp(-\Delta U / k_B T)$$

Quantum escape (tunneling) rate:

$$\Gamma = \text{independent of } T$$

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Measurements of Macroscopic Quantum Tunneling out of the Zero-Voltage State of a Current-Biased Josephson Junction

Michel H. Devoret,[a] John M. Martinis, and John Clarke
Department of Physics, University of California, Berkeley, California 94720, and Materials and Molecular Research Division, Lawrence Berkeley Laboratory, Berkeley, California 94720
(Received 26 July 1985)

To express the experimental measurements of the escape rate in a way that is as independent as possible of the parameters of the junction, we introduce the "escape temperature" T_{esc} defined through the relation

$$\Gamma = (\omega_p/2\pi)\exp(-\Delta U/k_B T_{esc}). \qquad (3)$$

Saturation due to quantum tunneling

Classical prediction

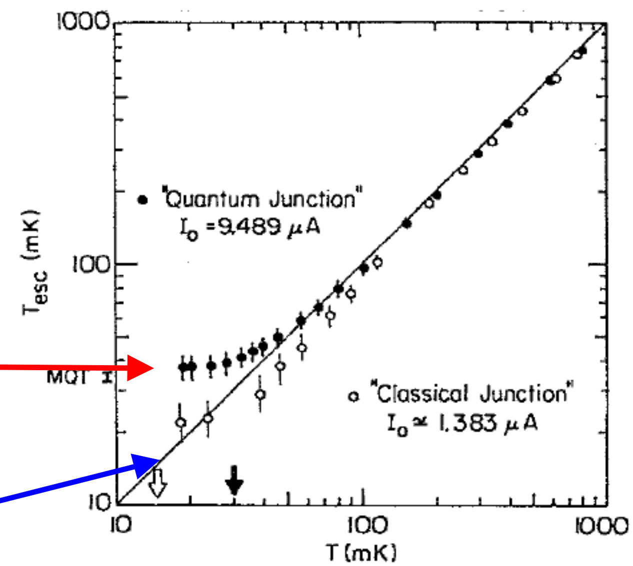
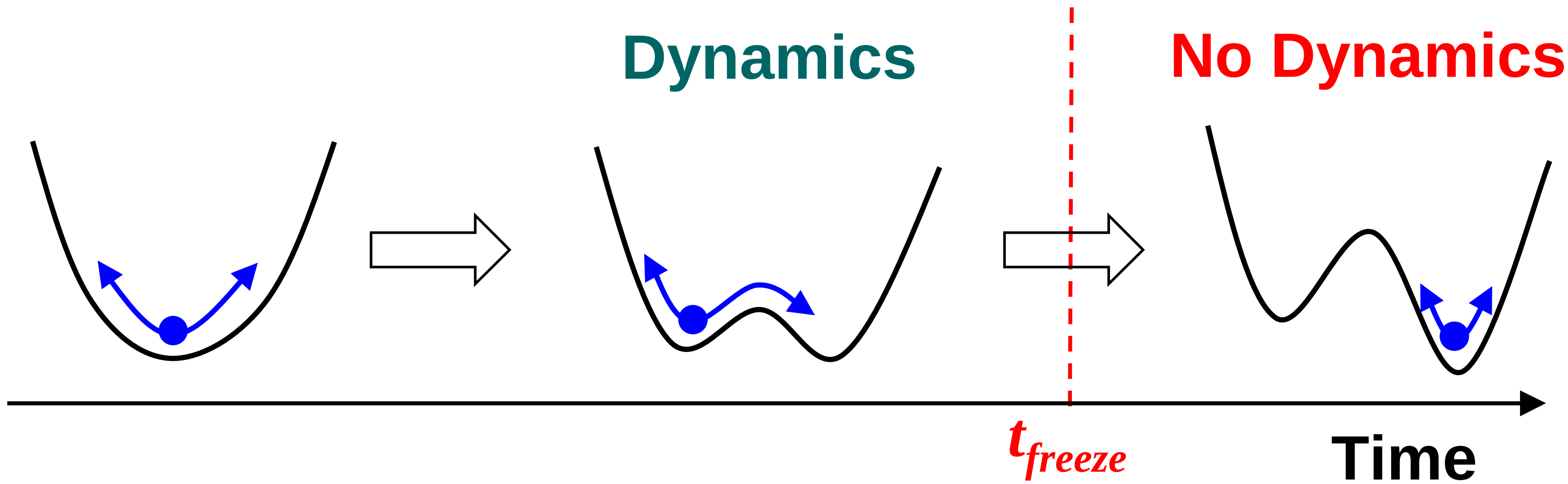


FIG. 2. T_{esc} vs T for two values of critical current for $\ln(\omega_p/2\pi\Gamma) = 11$. The solid and open arrows indicate the predicted crossover temperatures for the higher and lower critical currents, respectively. The prediction of Eq. (5) for the higher critical current is indicated at the left.

Classical Annealing

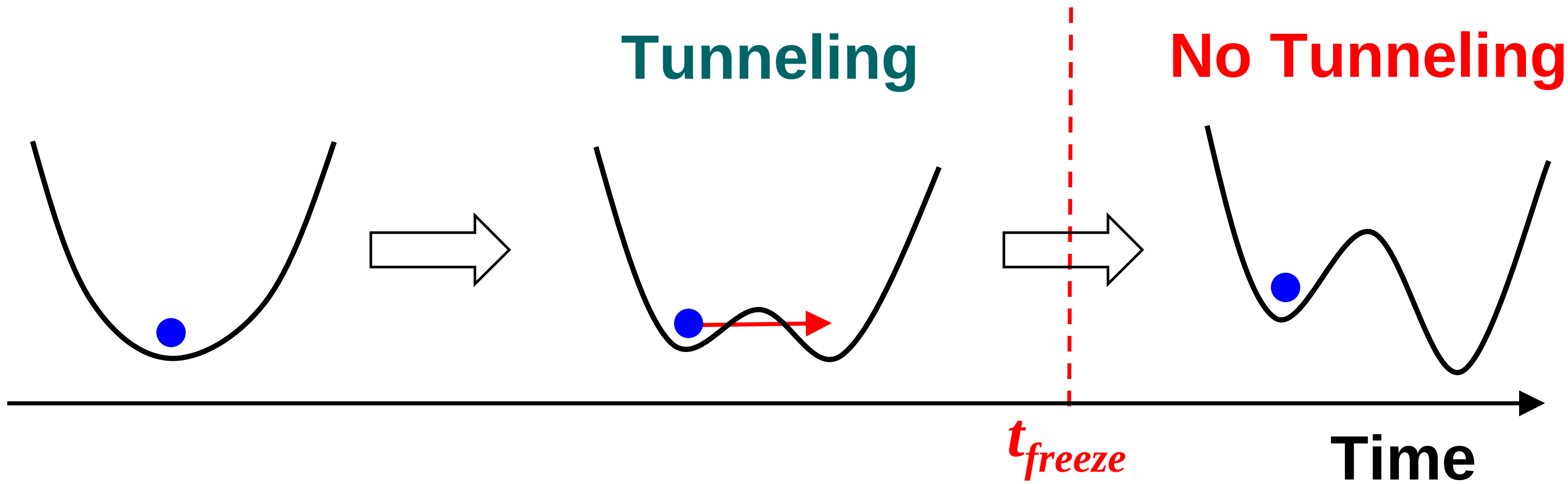


$$\Gamma = (\omega_p / 2\pi)\exp(-\Delta U_{freeze} / k_B T) = \text{ small number}$$

Therefore: $\Delta U(t_{freeze}) = \Delta U_{freeze} \propto T$

If $\Delta U(t) \propto t$ **then** $t_{freeze} \propto T$ **very similar to T_{esc}**

Quantum Annealing



Quantum mechanically tunneling amplitude exponentially depends on ΔU but not T

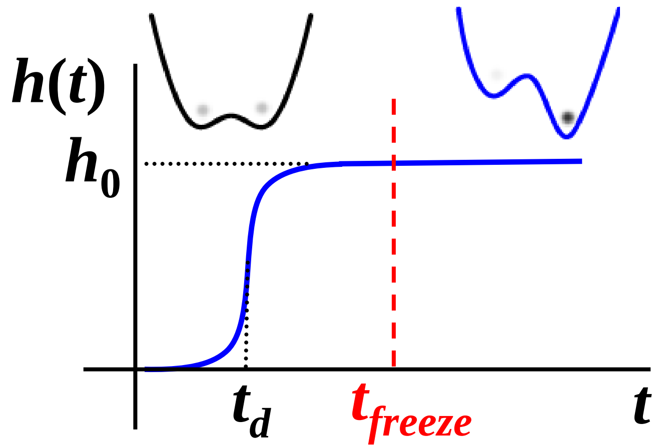
Therefore $\Gamma = T - \text{indepdent}$ **or**

$$t_{freeze} = T - \text{independent}$$

h-Ramping Experiment

$$\frac{H}{E(t)} = h(t)\sigma_z - \Gamma(t)\sigma_x$$

$$h(t) = \frac{1}{2}h_0\left[\tanh\left(\frac{t-t_d}{\delta t}\right)+1\right]$$



If $t_d < t_{freeze}$ then the final probability is: $P_0(h=h_0) > 0.5$

h-Ramping Experiment

$$\frac{H}{E(t)} = h(t)\sigma_z - \Gamma(t)\sigma_x$$

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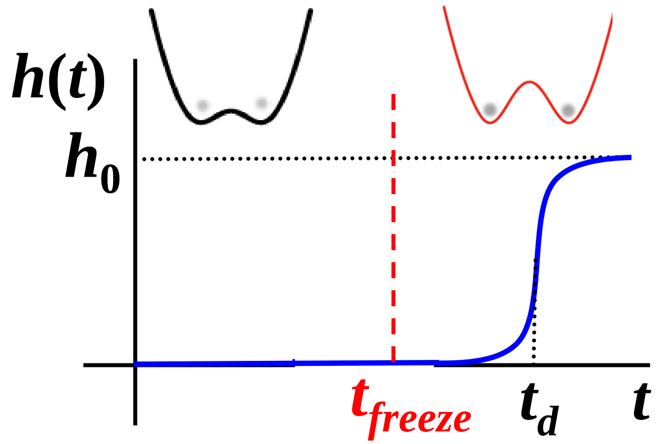
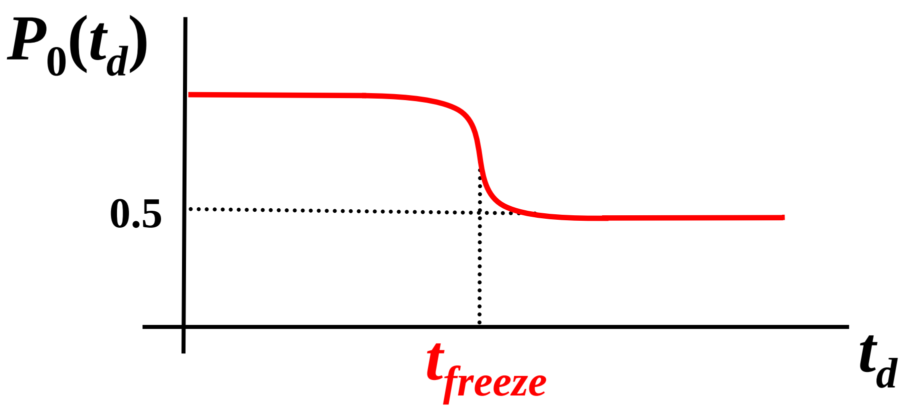


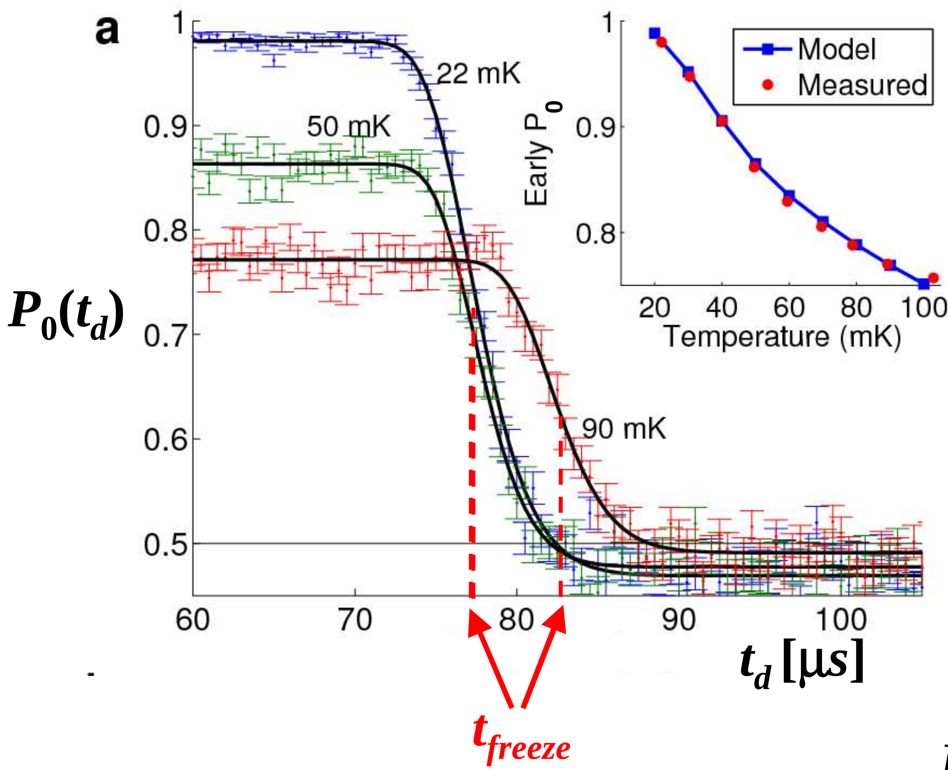
If $t_d < t_{freeze}$ then the final probability is: $P_0(h=h_0) > 0.5$

If $t_d > t_{freeze}$ then the final probability is: $P_0(h=0) = 0.5$

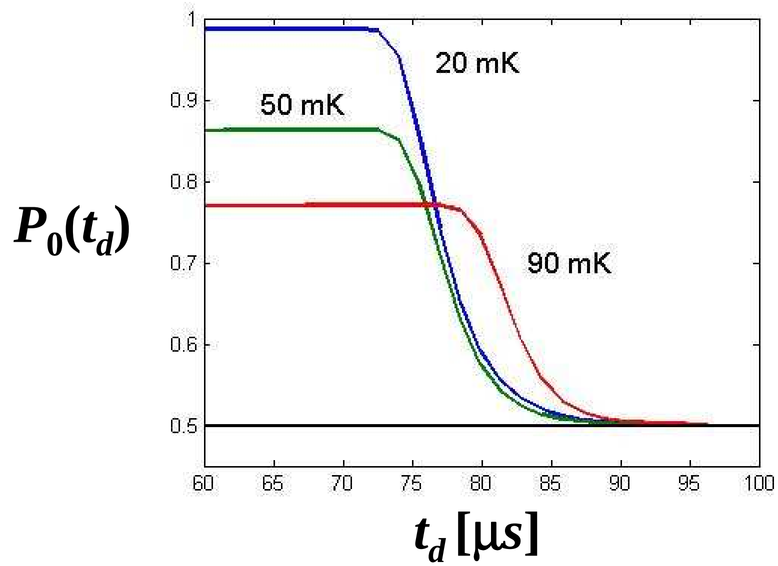


Single-Qubit Data

Experimental Data

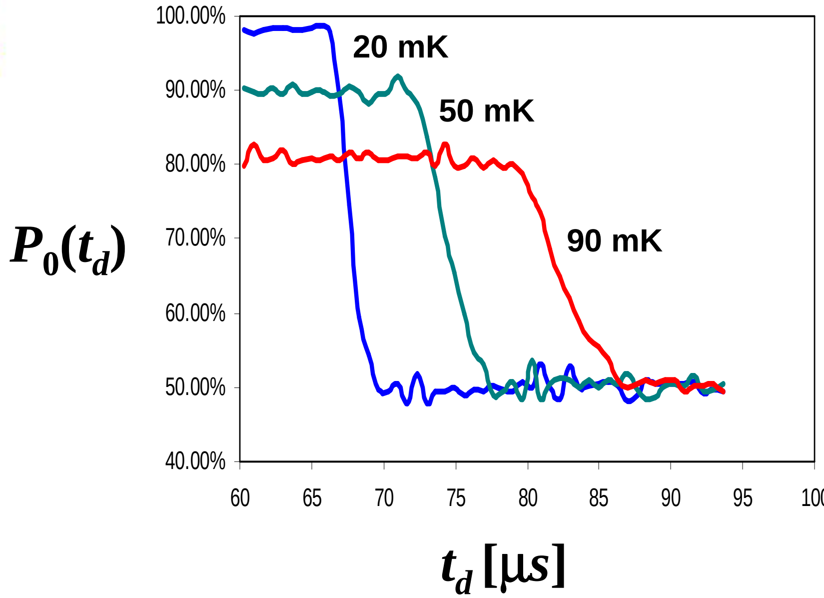


Quantum Simulations



Temperature is measured independently using MRT

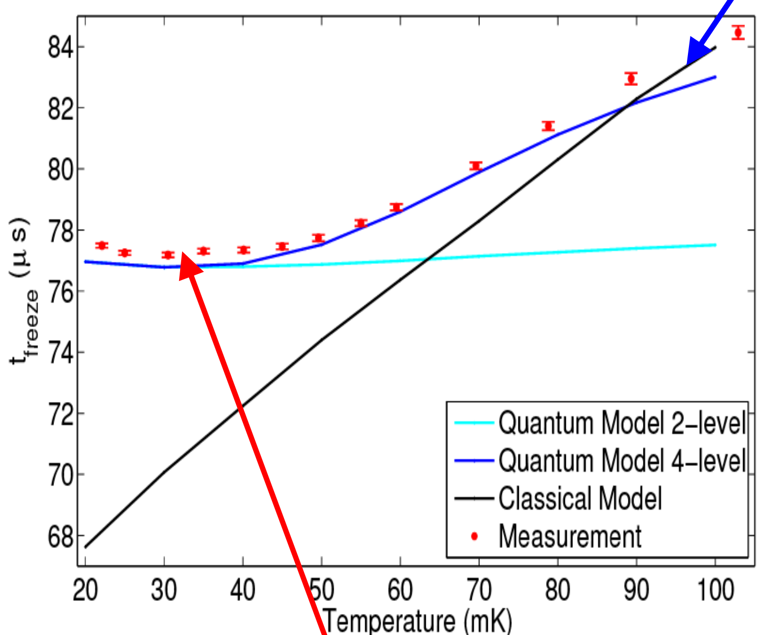
Classical Simulations



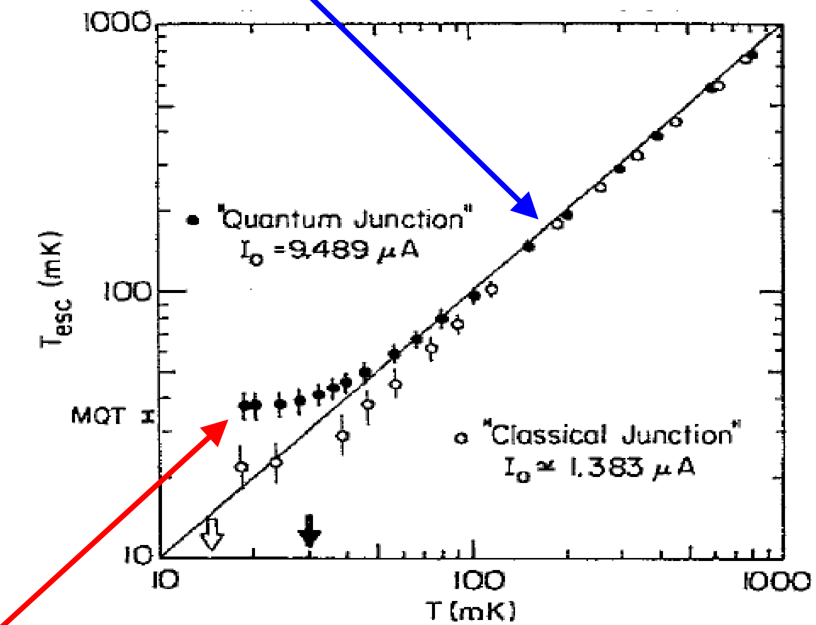
Freeze-out Time

Classical prediction

Freeze-out time measurement

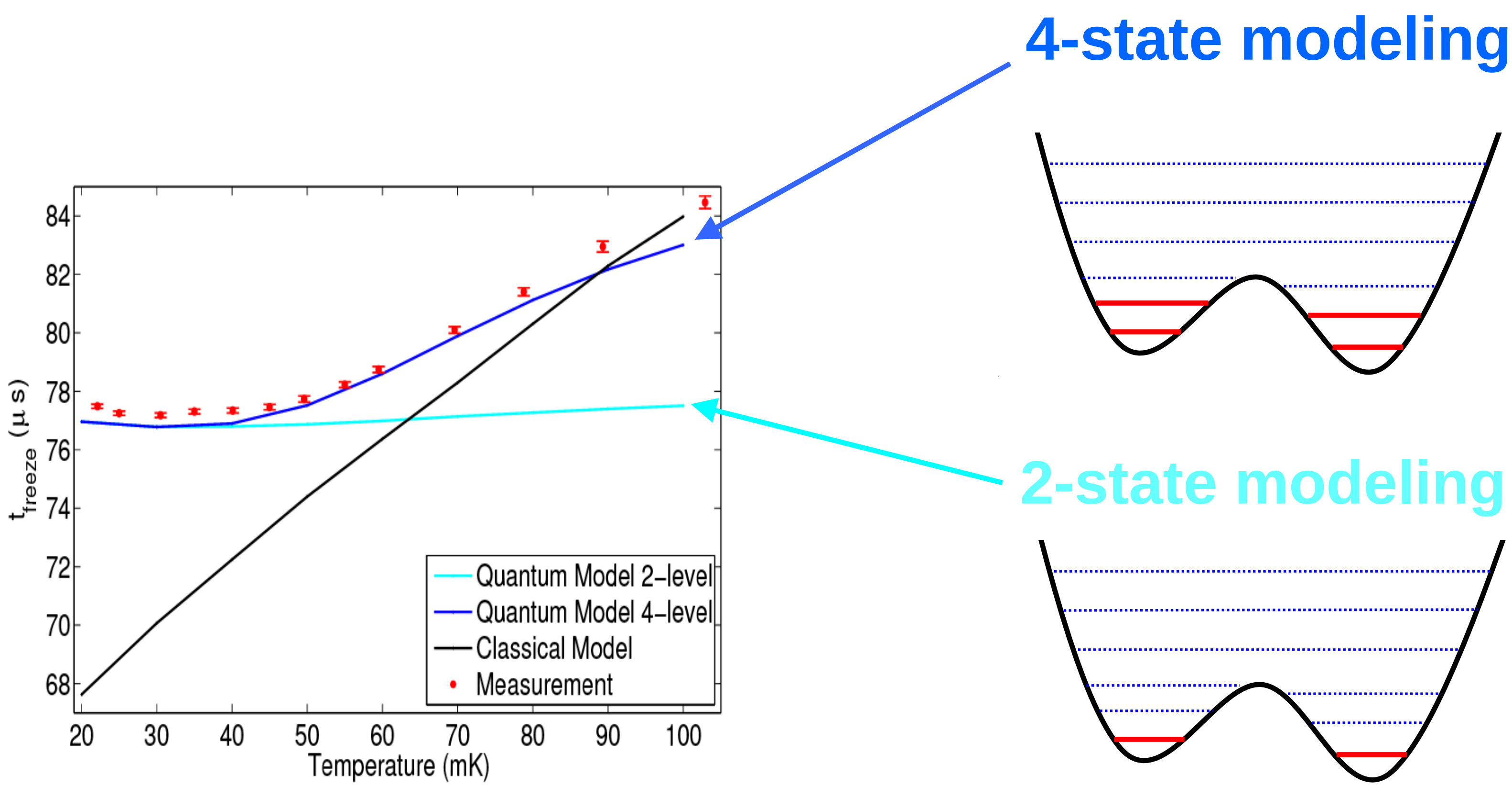


Devoret et al., PRL (1985)



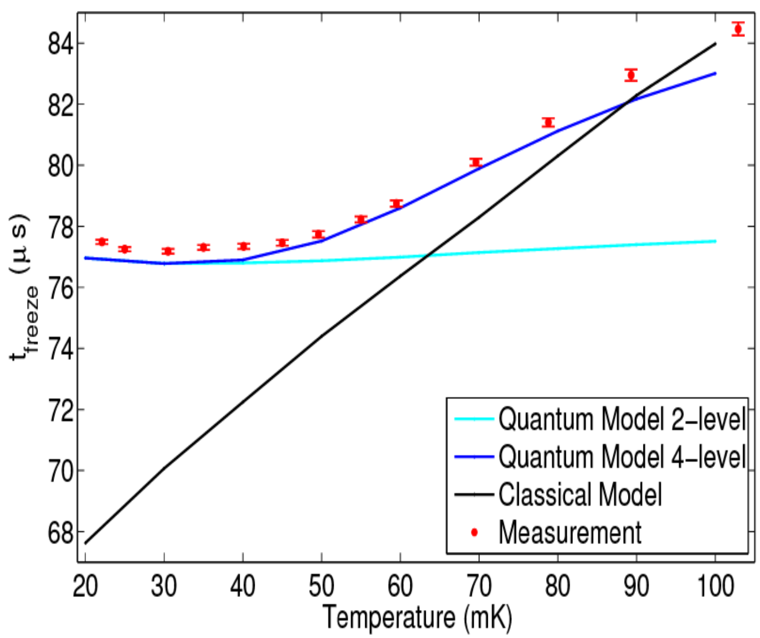
Saturation due to quantum tunneling

Experiment vs. Simulations



All simulations are done with no fitting parameter

Experiment vs. Simulations



rf-SQUID behavior at different temperatures:

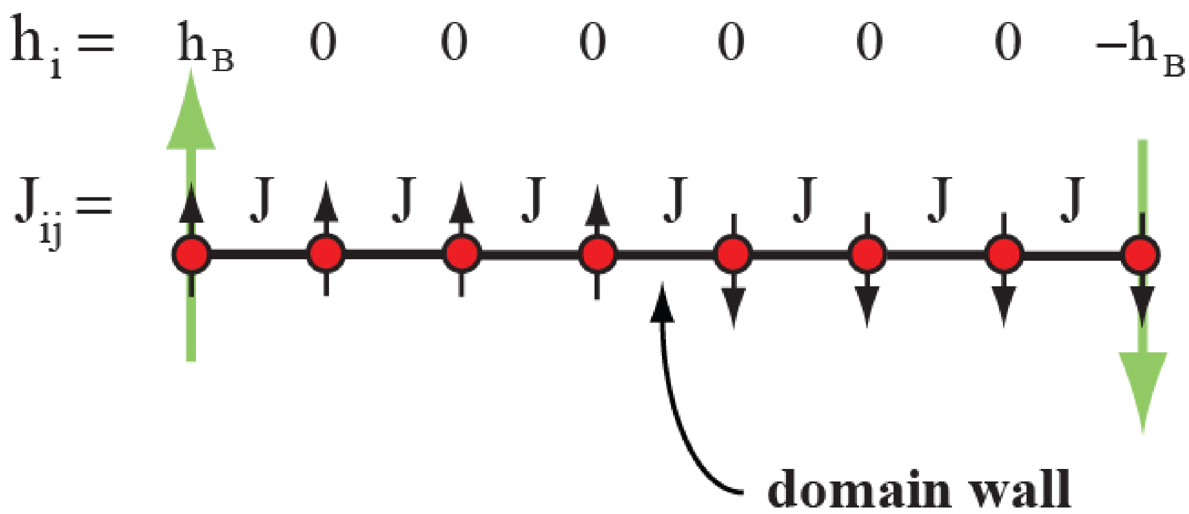
Up to 40 mK
2-level quantum system

Up to 80 mK
4-level quantum system

Above 100 mK
Classical system

Can we do this for more than one qubit?

8-Qubit Ferromagnetic Chain

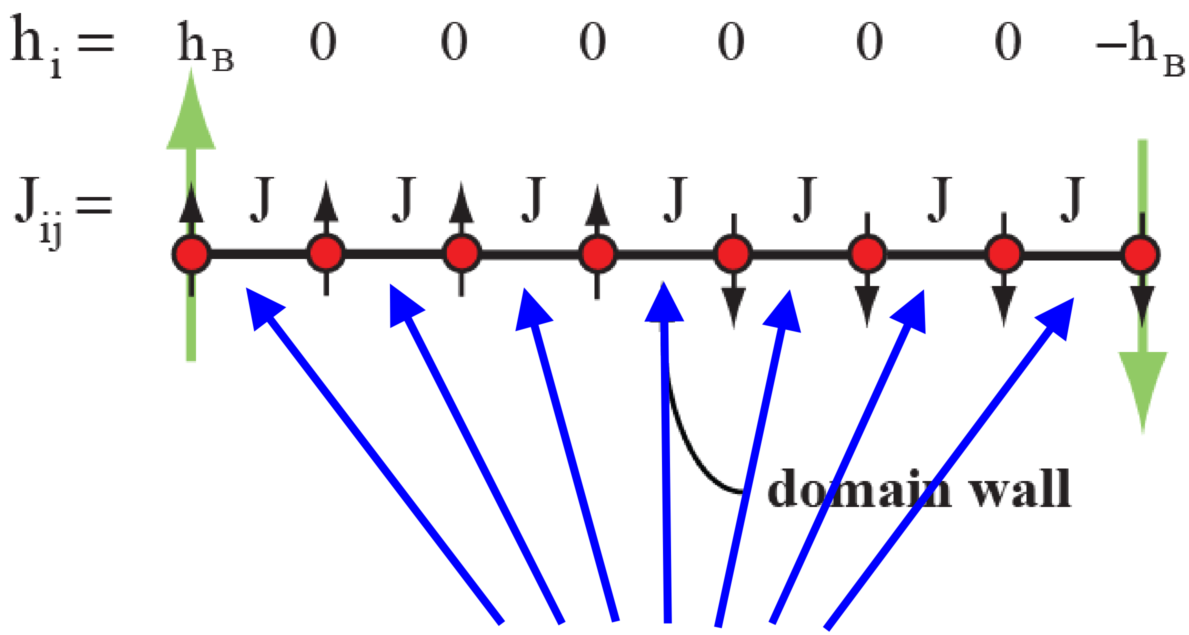


Hamiltonian:

$$\frac{H}{E(t)} = H_P - \Gamma(t)\sum_i \sigma_i^x$$

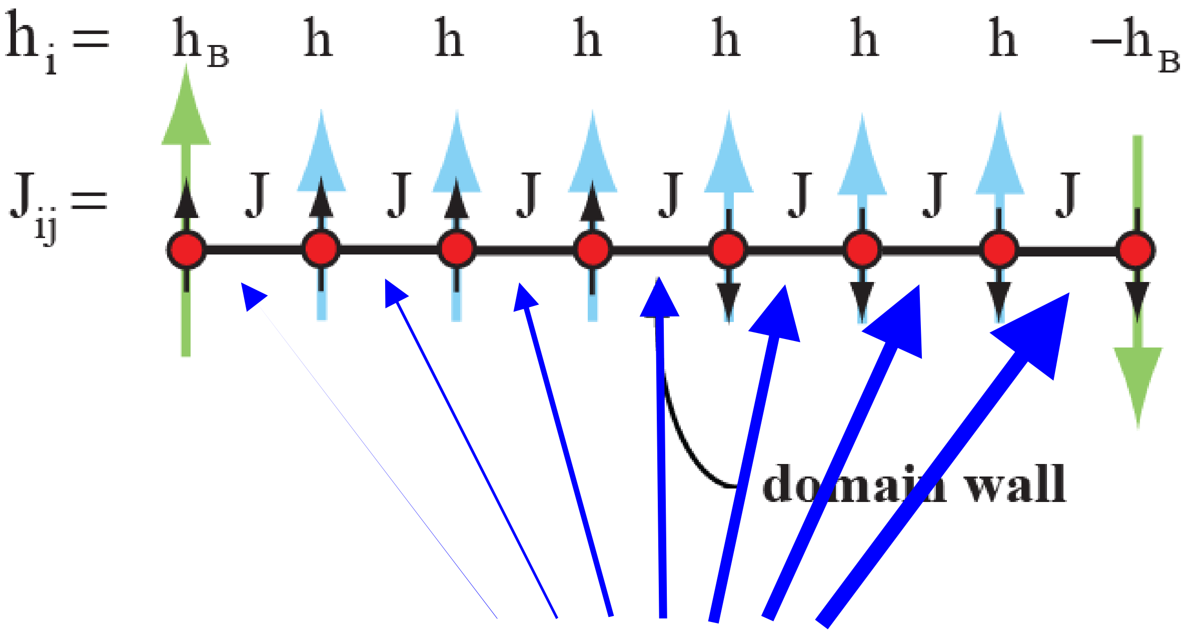
$$H_P = \sum_{i=1}^{N} h_i \sigma_i^z - J\sum_{i,j=1}^{N} \sigma_i^z \sigma_j^z$$

8-Qubit Ferromagnetic Chain



Domain wall can be in any of the 7 sites with probability 1/7

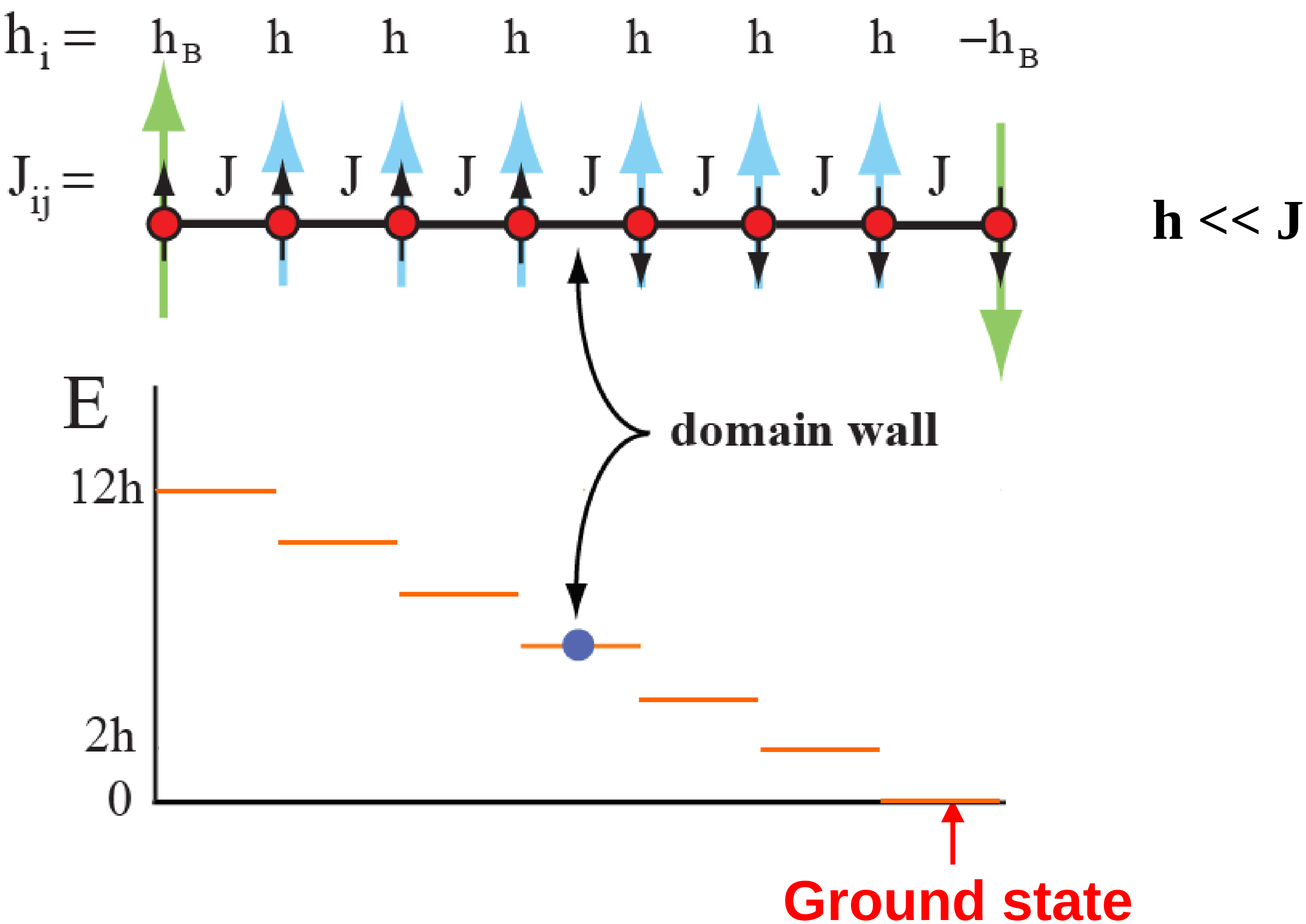
8-Qubit Ferromagnetic Chain



h << J

Domain wall most probably on the right site

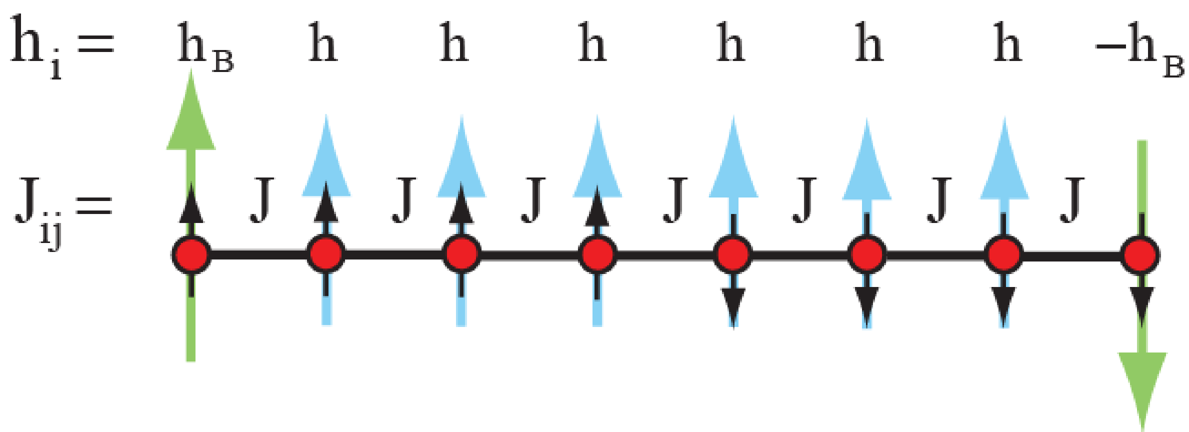
8-Qubit Ferromagnetic Chain



Question:

Is the ground state reached via classical or quantum annealing?

8-Qubit h-Ramping Experiment



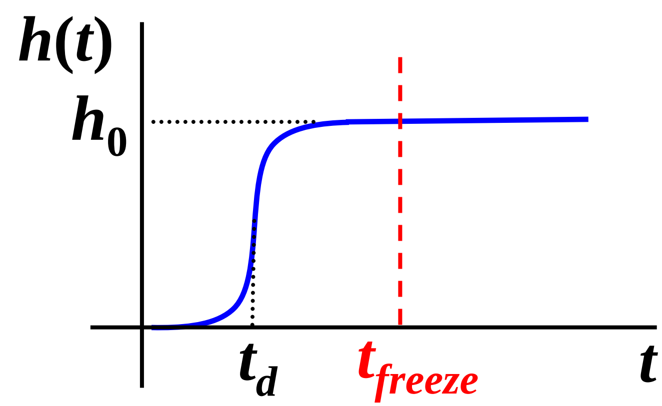
h << J

$$\frac{H}{E(t)} = H_P(h) - \Gamma(t)\sum_i \sigma_i^x$$

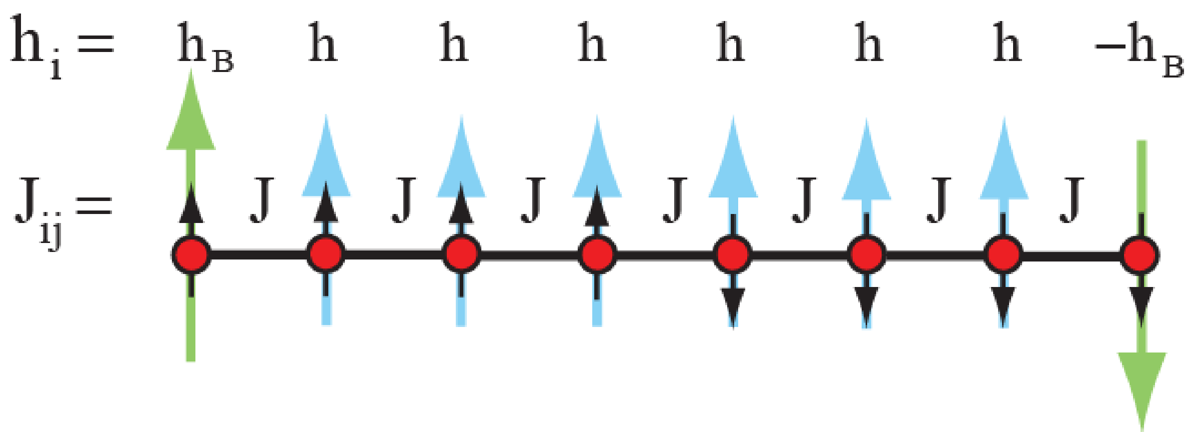
$$h(t) = \frac{1}{2}h_0\left[\tanh\left(\frac{t-t_d}{\delta t}\right)+1\right]$$

$t_d < t_{freeze}$

$P_0(h=h_0) > 1/7$



8-Qubit h-Ramping Experiment



h << J

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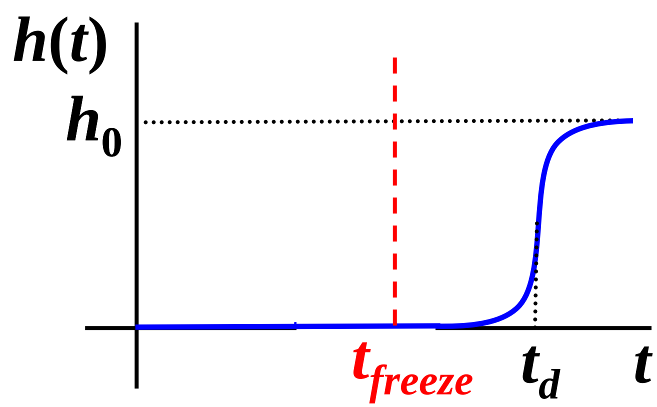
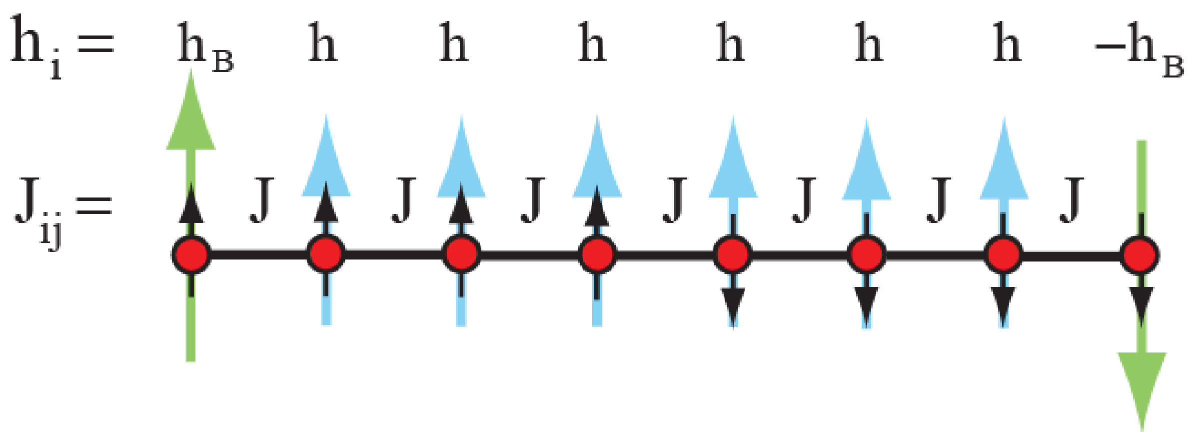
$t_d < t_{freeze}$ $\quad P_0(h=h_0) > 1/7$

$t_d > t_{freeze}$ $\quad P_0(h=0) = 1/7$



8-Qubit h-Ramping Experiment



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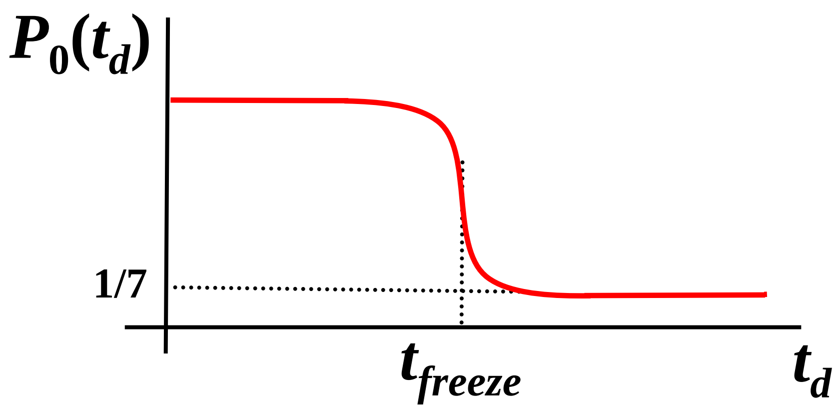
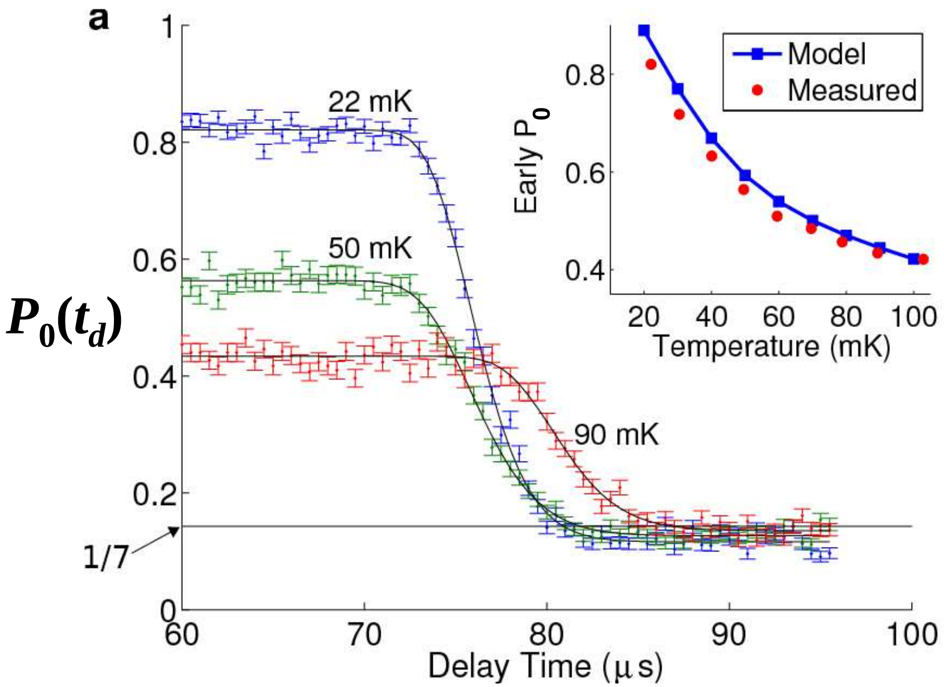
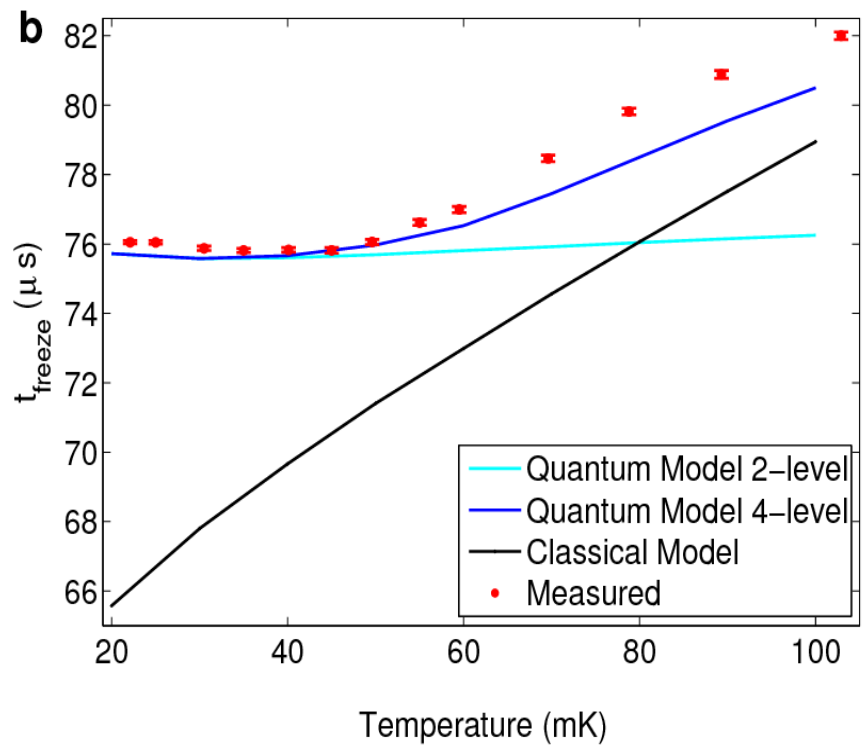
$t_d < t_{freeze}$ $P_0(h=h_0) > 1/7$

$t_d > t_{freeze}$ $P_0(h=0) = 1/7$



8-Qubit Experimental Results





Saturation of the freeze-out time cannot be explained by classical physics

Modeling The System

Write down a Hamiltonian and solve it both

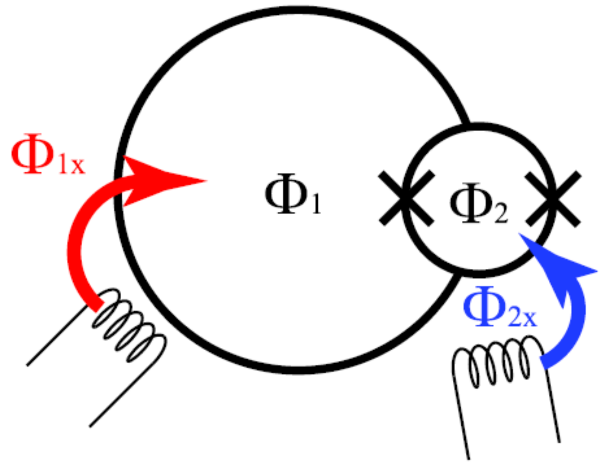
Classically:

Using Langevin equation

Quantum Mechanically:

Using density matrix approach

RF-SQUID Hamiltonian



Hamiltonian:

$$H_{\text{SQUID}} = \frac{q_1^2}{2C_1} + \frac{q_2^2}{2C_2} + U(\Phi_1, \Phi_2)$$

$$U(\Phi_1, \Phi_2) = (\Phi_1 - \Phi_{1x})^2/2L_1 + (\Phi_2 - \Phi_{2x})^2/2L_2$$
$$-2E_J \cos(\pi\Phi_2/\Phi_0)\cos(2\pi\Phi_1/\Phi_0),$$

Interaction with environment: $$H_{\text{int}} = -\frac{\Phi_1 - \Phi_{1x}}{L_1}\delta\Phi_{1x}$$

Noise spectral density:

$$S_{LF}(\omega) = \frac{(A^2/T)\omega|\omega|^{-\alpha}}{1 - e^{-\omega/T}} \approx A^2|\omega|^{-\alpha} \qquad S_{HF}(\omega) = \frac{\eta\omega e^{-|\omega|/\omega_c}}{1 - e^{-\omega/T}}$$

All parameters, including noise, are determined via independent experiments

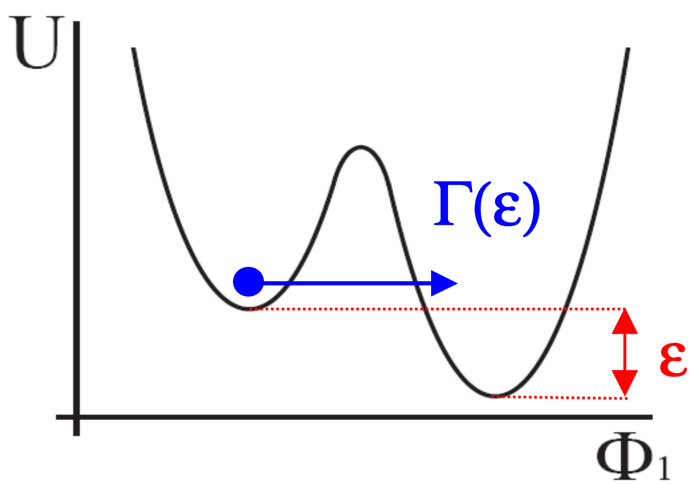
Extracting Noise Parameters

$$S_{LF}(\omega) = \frac{(A^2/T)\omega|\omega|^{-\alpha}}{1 - e^{-\omega/T}} \approx A^2|\omega|^{-\alpha}$$

$$S_{HF}(\omega) = \frac{\eta\omega e^{-|\omega|/\omega_c}}{1 - e^{-\omega/T}}$$

A is determined from 1/f noise measurement

η is extracted from measuring the Macroscopic Resonant Tunneling (MRT)

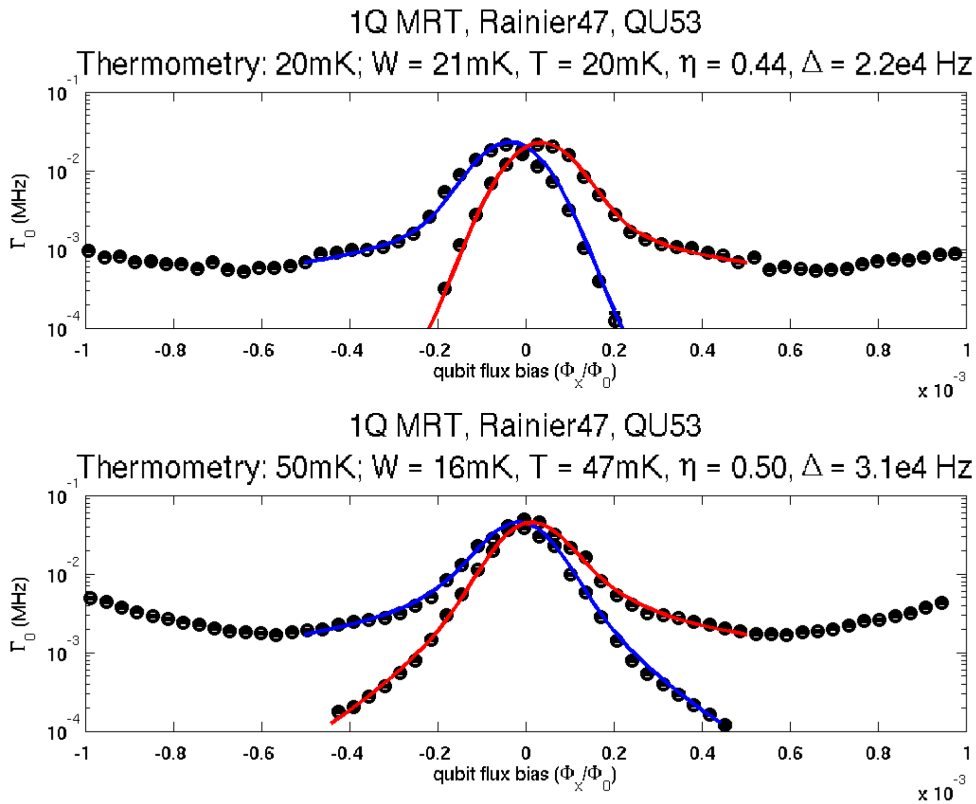


Extracting Noise Parameters

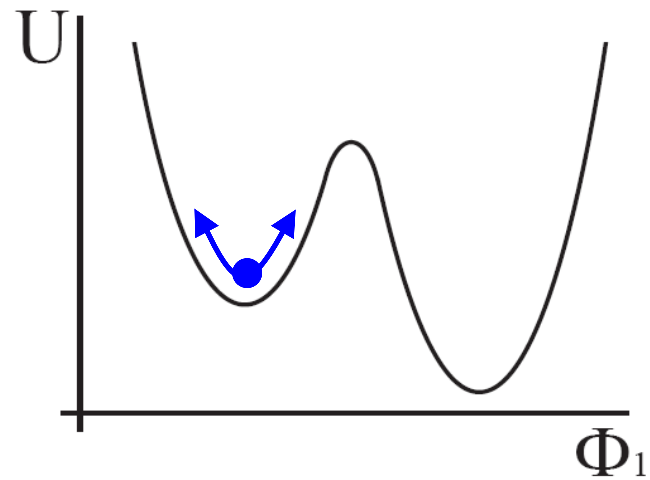
$$\Gamma(\epsilon) = \frac{\Delta^2}{4} \int dt e^{i(\epsilon - \epsilon_p)t - W^2 t^2/2} \left[\frac{\pi T t}{(1 + i\omega_c t) \sinh \pi T t} \right]^{\frac{\eta}{2\pi}}$$

$$W^2 = \int \frac{d\omega}{2\pi} S_{LF}(\omega)$$

$$\epsilon_p = \int \frac{d\omega}{2\pi} \frac{S_{LF}(\omega)}{\omega}$$



Classical Modeling



Langevin equation:

Force

$$\frac{dv}{dt} = \frac{F(x)}{\mathcal{M}} - \gamma v + X(t)$$

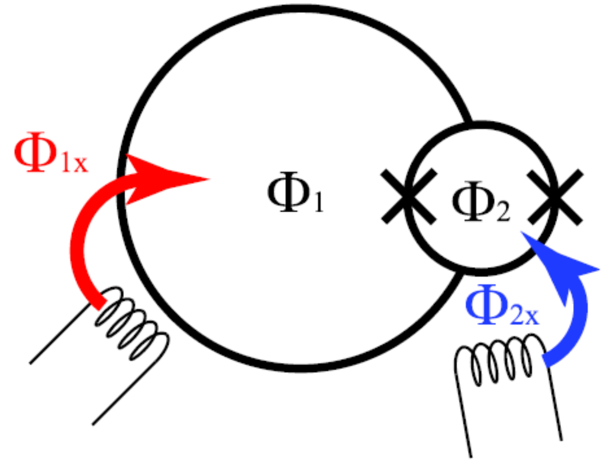
Dissipation **Noise**

$$x = 2\pi \frac{\Phi_1}{\Phi_0}$$

$$F(x) = -\frac{dU}{dx}$$

Fluctuation dissipation theorem: $\langle X(t) X(0) \rangle = \frac{2\gamma k_B T}{\mathcal{M}} \delta(t)$

Classical Modeling



Each rf-SQUID potential has 2 space variables (x_1,x_2) and 2 velocities (v_1,v_2)

For 8 qubits, we solve Langevin equation on a 16 dimensional potential

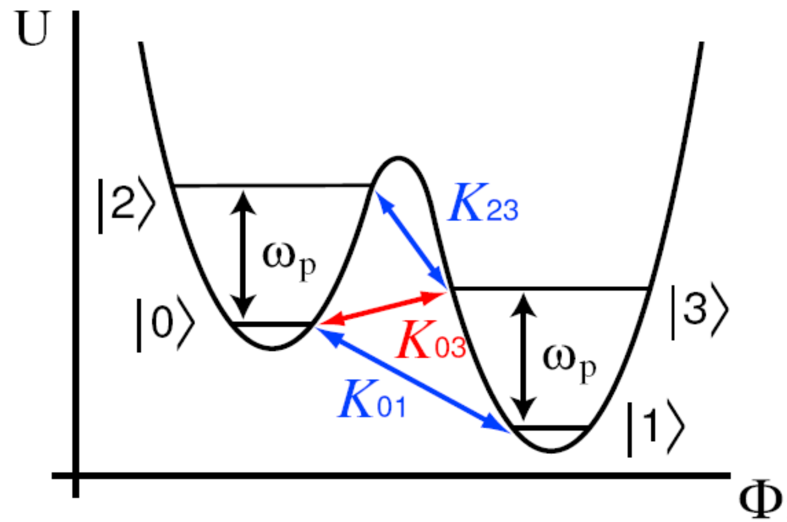
We run the program for more than 2000 times and find the probabilities statistically

We use distributed computation (AQUA@home):

~10,000 cores (>5500 computers in 78 countries)

~2,000,000 CPU core hours (>200 years)

Quantum Modeling



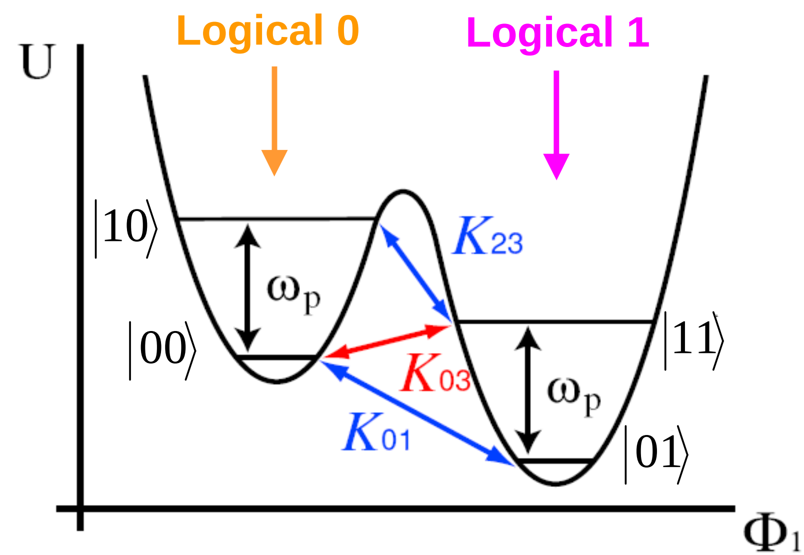
Quantum mechanically energy is quantized

$$H_S = \sum_{l=0}^{M-1} E_l |l\rangle\langle l| + \sum_{n,m=0}^{M/2-1} K_{2n,2m+1}(|2n\rangle\langle 2m+1| + |2m+1\rangle\langle 2n|)$$

Qubit Model: Keep lowest 2 energy levels

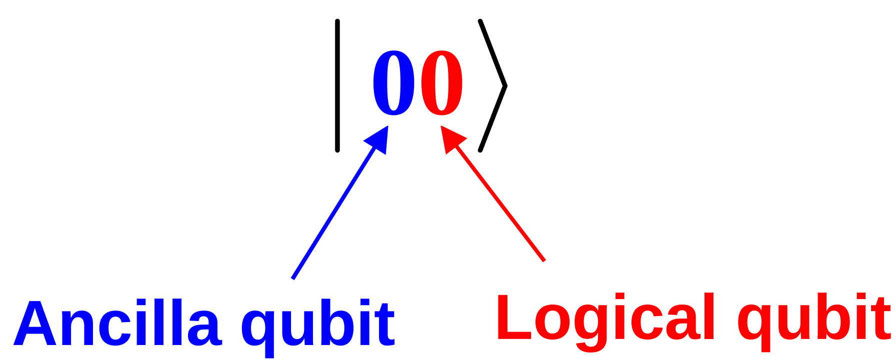
Qudit Model: Keep lowest d energy levels

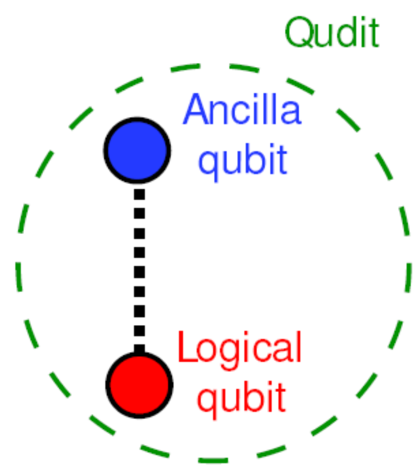
4-Level Qudit Model



All levels within each well are logically equivalent

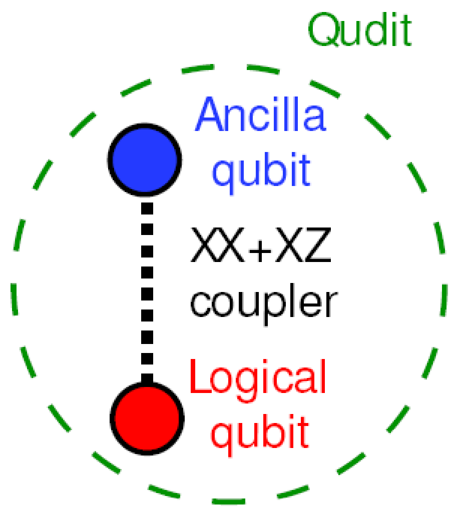
$$H_S = \sum_{l=0}^{M-1} E_l |l\rangle\langle l| + \sum_{n,m=0}^{M/2-1} K_{2n,2m+1}(|2n\rangle\langle 2m+1| + |2m+1\rangle\langle 2n|)$$





Qubit Representation of Qudits

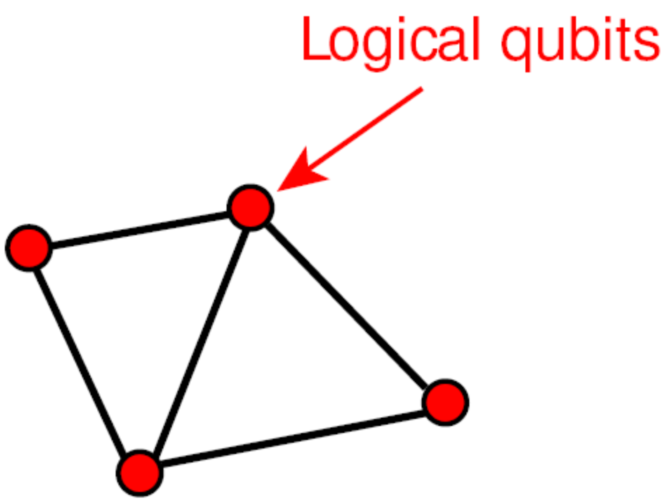
$$\begin{aligned} \epsilon &= E_1 - E_0 = E_3 - E_2, \\ \omega_p &= E_2 - E_0 = E_3 - E_1, \\ \Delta &= 2K_{01}, \\ \kappa_{xz} &= K_{23} - K_{01} \approx K_{23}, \\ \kappa_{xx} &= 2K_{03} = 2K_{12}. \end{aligned}$$



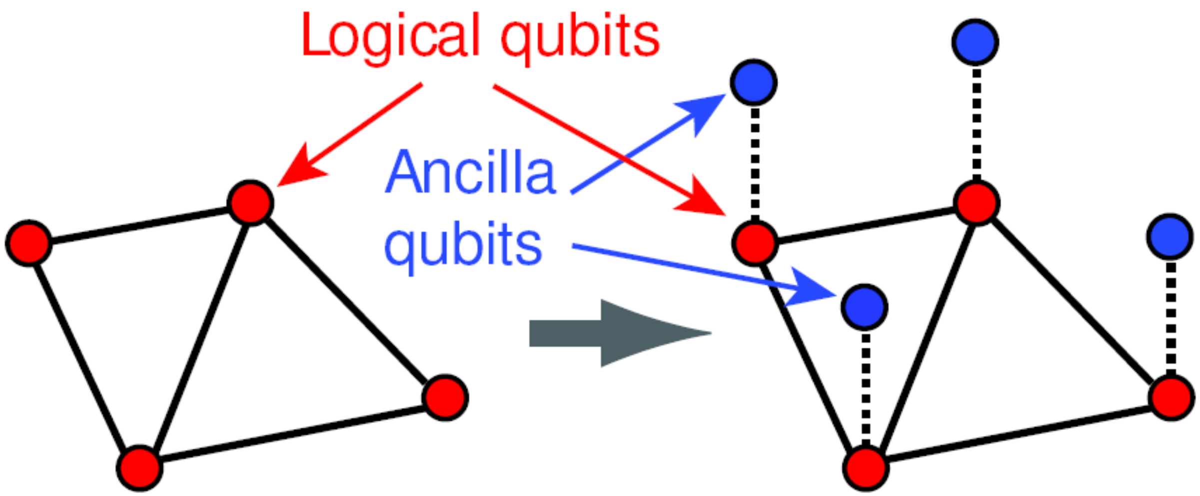
$$H_{eff} = \frac{1}{2}(\epsilon\sigma_z + \Delta\sigma_x) + \frac{1}{2}[\omega_p\tau_z + \kappa_{xz}\sigma_x(1+\tau_z) + \kappa_{xx}\sigma_x\tau_x]$$

XZ coupling **XX coupling**

Multi-Qubit System



Multi-Qubit System



Couple to each qubit an ancilla qubit with XX+XZ coupling

Multi-Qubit System

Coupled system Hamiltonian:

$$H_S = \frac{\Delta(t)}{2}\sum_{i=1}^{N}\sigma_x^{(i)} + E(t)\left[\sum_{i=1}^{N} h_i\sigma_z^{(i)} + \sum_{i,j=1}^{N} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}\right]$$

$$+\frac{1}{2}\sum_{i=1}^{N}\left[\omega_p^{(i)}(t)\tau_z^{(i)} + \kappa_{xz}^{(i)}(t)\sigma_x^{(i)}(1+\tau_z^{(i)}) + \kappa_{xx}^{(i)}(t)\sigma_x^{(i)}\tau_x^{(i)}\right]$$

Original qubit Hamiltonian

Interaction Hamiltonian:

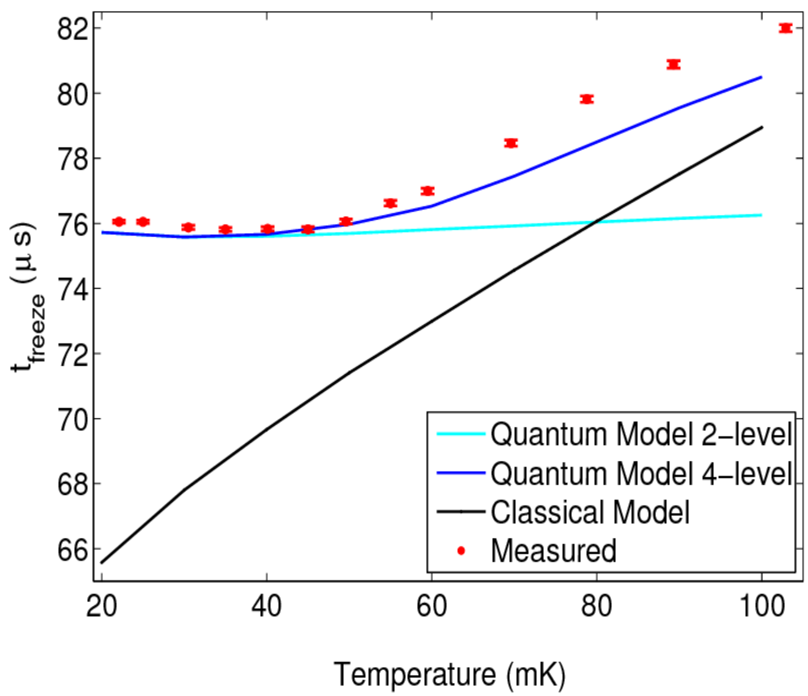
$$H_{\text{int}} = -\frac{1}{2}\sum_{i=1}^{N}(\sigma_z^{(i)} + \lambda\tau_x^{(i)})Q_i$$

All parameters are directly extracted from the rf-SQUID Hamiltonian

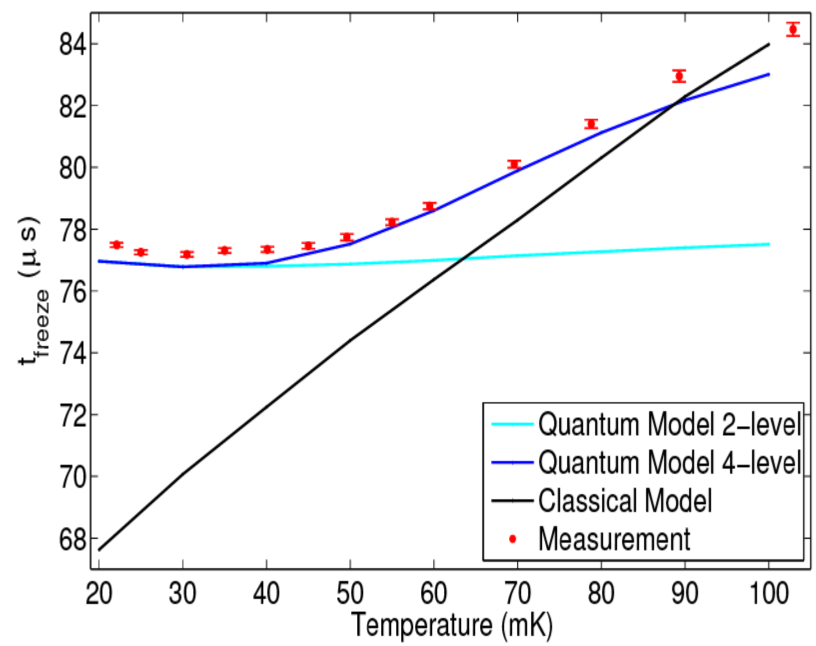
The final ground state is unaffected by ancilla qubits

Experiment vs. Simulations

8 Qubit Chain



Single Qubit



Classical and quantum simulations agree with experiment with no free parameters

Collaborators:

Experiment:

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