Quantum Error Correction

Lecture 4

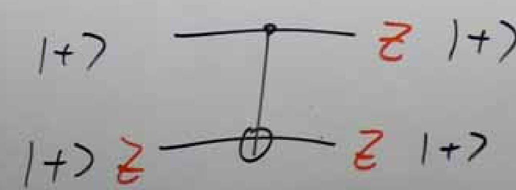
•

Daniel Gottesman, Perimeter Institute

University of British Columbia, July 18, 2010

Fault-Tolerance:

- Perform operations on states encoded in a QECC without losing protection against errors.
- Control error propagation



For a stabilizer code:

$N(S) \setminus S$ maps codewords to <u>different</u> codewords

Tensor products of single-qubit operations – no error propagation

$N(S) \setminus S$ can be used to implement logical Pauli group in a fault-tolerant way.

$N(S)/S \cong \mathcal{P}_k$

for a full FT protocol:

- FT gates (a universal set)
- Prepare encoded states in a FT way.
- FT measurement
- FT error correction

7-qubit code:

$$|\bar{0}\rangle = \sum_{\text{even } x \in C} |x\rangle$$

$$|\bar{1}\rangle = \sum_{\text{odd } x \in C} |x\rangle$$

Logical X: $X \otimes X \otimes X \otimes X \otimes X \otimes X \otimes X$

Logical Z: $Z \otimes Z \otimes Z \otimes Z \otimes Z \otimes Z \otimes Z$

Logical Hadamard H:

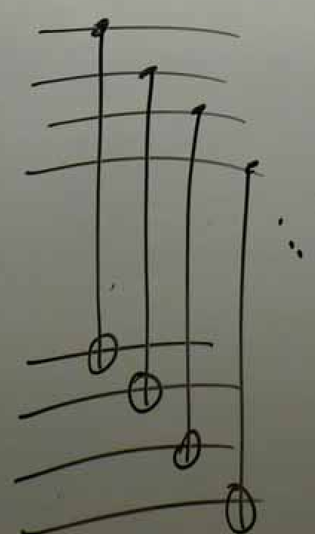
$$H \otimes H \otimes H \otimes H \otimes H \otimes H \otimes H$$

Logical $\pi/4$ phase $R_{\pi/4} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$:

$$R_{\pi/4}^{-1} \otimes R_{\pi/4}^{-1} \otimes R_{\pi/4}^{-1} \otimes R_{\pi/4}^{-1} \otimes R_{\pi/4}^{-1} \otimes R_{\pi/4}^{-1} \otimes R_{\pi/4}^{-1}$$

Transversal gates

Interacts ith qubit of one block with ith qubit of second block

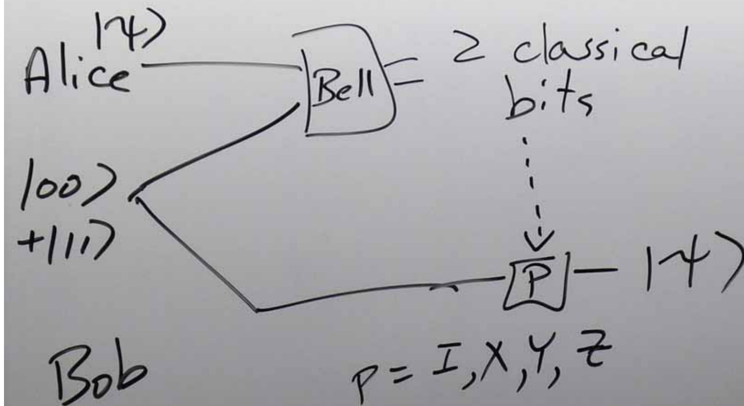


Logical CNOT:

$CNOT^{\otimes 7}$

Errors propagate – but not within a block.

Quantum Teleportation:

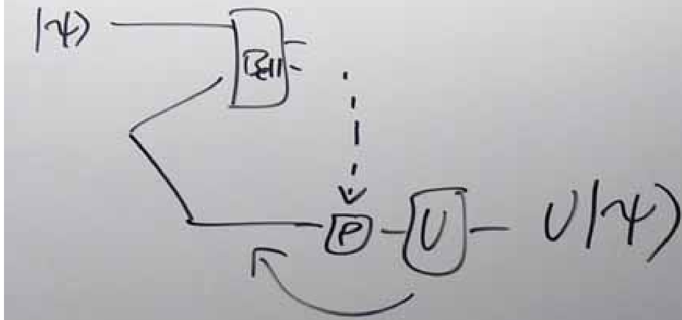


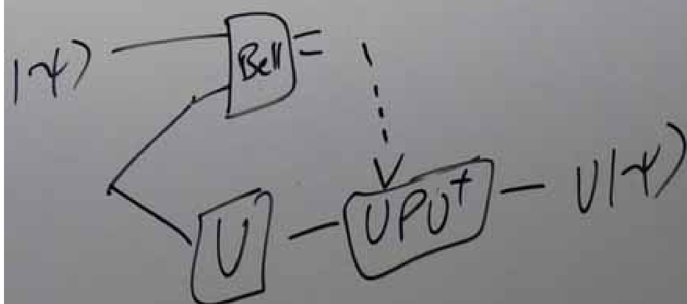
Bell measurement:

Projector on

$$|00\rangle \pm |11\rangle$$

$$|01\rangle \pm |10\rangle$$

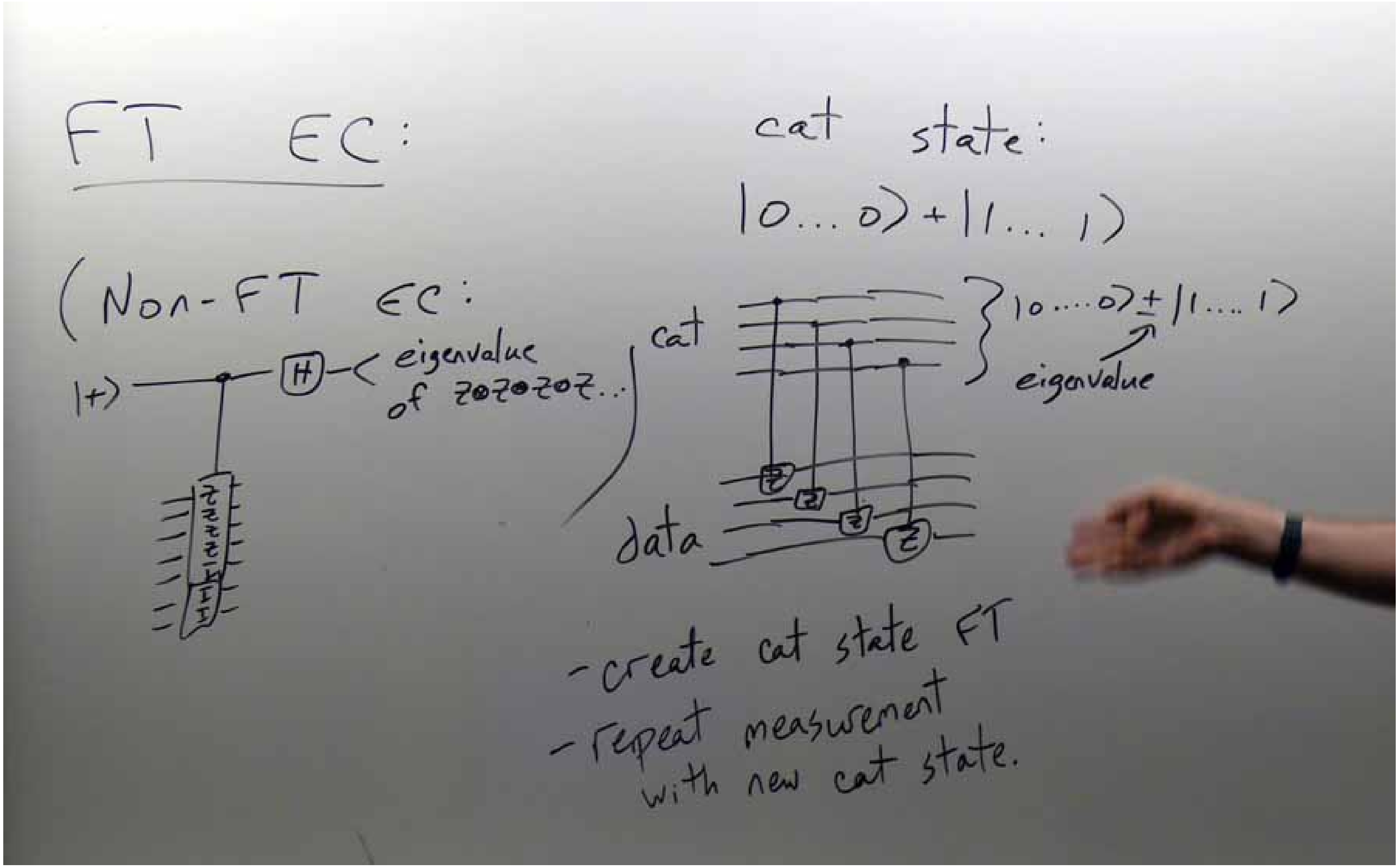




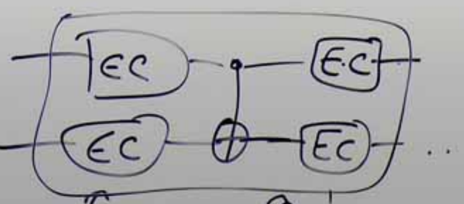
Encoded Ancilla

$|00\rangle + |11\rangle$ — U — $= (I \otimes U)(|00\rangle + |11\rangle)$

Encoded Teleport using this ancilla – provided we can perform $UPU^{\dagger}$ FT then can do gate U. FT


FT EC:
(Non-FT EC:
|+⟩
H
eigenvalue of Z⊗Z⊗Z⊗Z...
Z
I
cat state:
|0...0⟩+|1...1⟩
cat
data
|0....0⟩±|1....1⟩
eigenvalue
-create cat state FT
-repeat measurement with new cat state.

Imagine error rate p
per gate or time step



many physical gates
A in total

If there are two errors in here, then we have a logical error

If there's 0 or 1 error, we are OK

Prob. (≥ 2 errors)

$$\approx \binom{A}{2} p^2$$

If this is less than p, then the logical gates are more reliable than unencoded gates

$$\underbrace{\binom{A}{2} p^2}_{p^2/p_T} < p \Rightarrow p < \frac{1}{\binom{A}{2}} = p_T$$

Threshold Theorem:

- If $p < p_T$, then we can perform arbitrarily long quantum computations with polylogarithmic overhead.

Proof: Use concatenated codes - encode code state multiple times using a QECC

$$p/p_T \to (p/p_T)^2 \to (p/p_T)^4 \to (p/p_T)^8 \to \dots \to (p/p_T)^{2^L} \text{ with } L \text{ levels.}$$

Overhead c^L physical qubits per logical qubit

T logical gates $\Rightarrow$ logical error rate $\sim 1/T \Rightarrow L = \log\log T \Rightarrow$ overhead $\mathrm{poly}(\log T)$

Value of p_T:

Best proofs $p_T \gtrsim 10^{-3}$

Simulations $p_T \sim 5\%$